المراجعة رقم (1)

الترم الاول

Mathematics
second secondary first term 2020

1 Choose The correct answer

$\lim_{x \to \infty} \frac{x^3+5}{x(2x^2+3)} = \ldots$

a) $\frac{5}{8}$ b) 1 c) $\frac{1}{2}$ d) $\frac{5}{3}$

2 In The triangle ABC, if $4\sin A = 3\sin B = 6\sin C$, Then $m(\angle C) = \ldots$

a) 89 b) 29 c) 57 d) 82

3 If the function f where $f(x) = \begin{cases} \frac{x^2-1}{x-1} & , x \neq 1 \\ 2a & , x = 1 \end{cases}$

is continuous at $x = 1$, then $a = \ldots$

a) zero b) -2 c) 2 d) 1

4 In the triangle XYZ, the expression

$\frac{x^2+y^2-z^2}{2xy} = \ldots$

a) cos X b) cos Y c) cos Z d) sin Z

5 $\lim_{x \to 3} \frac{x-3}{x^2-9} = \ldots$

a) 3 b) $\frac{1}{9}$ c) $\frac{1}{3}$ d) $\frac{1}{6}$

6 In The triangle ABC, $\cos A = \ldots$

a) $\frac{a^2+b^2-c^2}{2ab}$ b) $\frac{a^2+c^2-b^2}{2ab}$ c) $\frac{b^2+c^2-a^2}{2bc}$ d) $\frac{c^2-a^2+b^2}{2ab}$

7 If $(\frac{1}{2})^{a^2-a-2} = 1$, $a > 0$, then $a = \ldots$

(a) 1 (b) -3 (c) 2 (d) 4

8 If $f(x) = x+2$, then $f^{-1}(x) = \ldots$

(a) $x+2$ (b) $-x+2$ (c) $x-2$ (d) $\frac{x}{2}$

9 If the curve of the function f where $f(x) = \log_a x$ passes through the point $(8,3)$ then $f(4) = \ldots$ (a) 1 (b) 2 (c) -4 (d) -2

10 From the following functions, the one-to-one function is ....

(a) $f(x) = x-3$ (b) $g(x) = x^2$

(c) $r(x) = |x|$ (d) $h(x) = -7$

11 The range of the function f: $f(x) = |x-2|+1$ is ....

(a) $[1, \infty[$, (b) $]1, \infty[$ (c) $]2, \infty[$ (d) $[2, \infty[$

12 If $f(x) = \log_{(x-2)}(x)$, then the domain of f is ...

(a) $[2, \infty[$ (b) $]2, \infty[- \{3\}$ (c) $\mathbb{R}^+$ (d) $]0, 2[$

13 The curve of the function g: $g(x) = |x+3|$ is the same curve of the function f: $f(x) = |x|$ by translation 3 units in the direction of ...

(a) $\overrightarrow{o\vec{x}}$ (b) $\overrightarrow{o\vec{x}'}$ (c) $\overrightarrow{o\vec{y}}$ (d) $\overrightarrow{oy'}$

14 If $f(x+2) = 5^x$, then $f(0) = \ldots$

(a) 25 (b) $\frac{1}{25}$ (c) 15 (d) 5

15 ABC is a triangle in which

$$\frac{\sin A}{3} = \frac{2\sin B}{5} = \frac{\sin C}{4}$$, then a : b : c : ...

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 6

16 $\lim_{x \to \infty} \frac{\sqrt{x^2+3}}{2x+1} = \ldots$

(a) 1 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) 3

17 $\lim_{x \to 0} \frac{x^2+x}{x} = \ldots$

(a) 1 (b) 0 (c) 2 (d) doesn't exist

18 If $f : f(x) = \begin{cases} ax^2 - 6, & x \neq 2 \\ 2a, & x = 2 \end{cases}$

is continuous at $x = 2$, then $a = \ldots$

(a) 4 (b) $\frac{3}{2}$ (c) 3 (d) $\sqrt{3}$

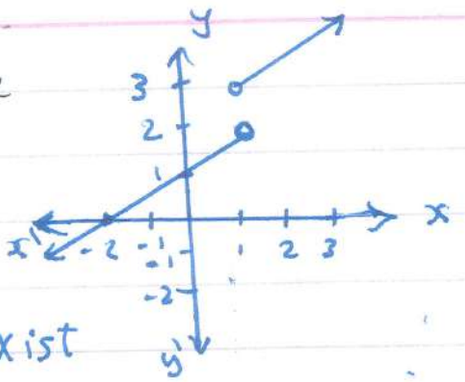
19 In ΔABC, if $2\sin A = 3\sin B = 4\sin C$, then $a : b : c = \ldots$

(a) 2 : 3 : 4 (b) 9 : 3 : 2 (c) 2 : 4 : 6 (d) 6 : 4 : 3

20 In ΔXYZ, if $x = 3$ cm, $y = 4$ cm, and $z = 6$ cm, then $\cos Z = \ldots$

(a) $\frac{-11}{24}$ (b) $\frac{24}{11}$ (c) $\frac{-11}{12}$ (d) $\frac{-12}{11}$

21 The opposite figure represents the graph of the function f, then $\lim_{x \to 1} f(x) = \ldots$



(a) 2 (b) 3

(c) 1 (d) not exist

22 $\lim_{x \to 0} \frac{1+x}{4x-1} = \ldots$

(a) -1 (b) $\frac{1}{4}$ (c) $-\frac{1}{4}$ (d) 1

23 $\lim_{x \to 1} \frac{x^7 - 1}{x - 1} = \ldots$

(a) 7 (b) 8 (c) 6 (d) zero

24 $\lim_{x \to \infty} \frac{x^{-3} + 3x^{-2} + 1}{x^{-2} + x^{-1} + 3} = \ldots$

(a) 2 (b) 1 (c) 3 (d) $\frac{1}{3}$

25 $\lim_{x \to 2} 2x \csc 4x = \ldots$

(a) 2 (b) 4 (c) $\frac{1}{2}$ (d) zero

26 If $f(x) = x^2$, then $\lim_{x \to 2} f(f(x)) = \ldots$

(a) 2 (b) 4 (c) 16 (d) 32

27 The function $f: f(x) = 4x^{-3} + \frac{x}{x^2 - 9}$ is continuous for every $x \in \ldots$

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - \{3, -3\}$ (d) $\mathbb{R} - \{3, -3, 0\}$

28 If the function f where $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ K, & x = 1 \end{cases}$ is continuous at $x = 1$, then $K = \ldots$ 5

(a) zero (b) -2 (c) 2 (d) 1

29 ABC is an equilateral triangle, the length of its side is $10\sqrt{3}$ cm, then the length of the diameter of its circumcircle is ... cm

(a) 20 (b) 10 (c) $5\sqrt{3}$ (d) $10\sqrt{3}$

30 $\lim_{x \to \infty} \frac{5x^{-3} + 4x^{-2} - 3}{7x^{-3} - 2x^{-2} + 8} = \ldots$

(a) $\frac{5}{7}$ (b) 2 (c) $\frac{3}{8}$ (d) $\frac{-3}{8}$

31 In the triangle XYZ, if $\frac{\sin X}{3} = \frac{\sin Y}{4} = \frac{\sin Z}{5}$,

Then the measure of the biggest angle in the triangle is ...

(a) 60° (b) 75° (c) 90° (d) 120°

32 $\lim_{x \to \pi} \frac{\sin x}{\pi - x} = \ldots$

(a) 1 (b) π^2 (c) π (d) $-\pi$

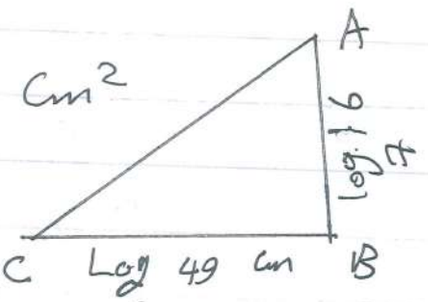
33 If $\lim_{x \to 2} \frac{a}{x+1} = 2$, then $a = \ldots$

(a) $\frac{2}{3}$ (b) 2 (c) 0 (d) 6

34 If $f(x) = 3x+1$ and $g(x) = x^2 - 5$, then $(fog)(2) = \ldots$ [-1 , -2 , 7 , 15]

35 If $f(x) = x+2$, then $f^{-1}(5) = \ldots$
[5 , -5 , 2 , 3]

36 In The opposite figure:
The area of the triangle = ... cm^2
[4 , 8 , 16 , 49]



37 The point of symmetry of the curve of the function f: $f(x) = (x+1)^3 - 2$ is ...
(a) $(1, 2)$ (b) $(-1, -2)$ (c) $(1, -2)$ (d) $(-1, 0)$

38 If $f(x) = 2x+1$, then $f^{-1}(x) = \ldots$

(a) $\frac{1}{2}x$ (b) $\frac{1}{2x+1}$ (c) $\frac{1}{2}(x+1)$ (d) $\frac{1}{2}(x-1)$

39 The S.S. of the equation: $|x+1|+3=0$ in $\mathbb{R}$ is ... [ϕ , $\{-1, -3\}$, $\{-3\}$, $\{3, -3\}$

40 The domain of the function f: $f(x) = \log(x+3)$ is ... [$-3, 2]$, $]-3, \infty[$, $]-\infty, \infty[$, $]3, \infty[$

41 If $f(x) = x+1$ and $g(x) = x^2$, then $(fog)(2) = \ldots$
[9 , 5 , 4 , 3]

42 The expression $\frac{3\log 2}{\log 4 + \log 3}$ is equivalent to

(a) $\log_{12} 8$ (b) $\log_4 2$ (c) $\log_3 2$ (d) $\log_7 8$

43) In ΔABC, if $a = 6$ cm and $m(\angle B) = 2m(\angle A) = 80^\circ$ then $C = \ldots$

(a) $\frac{4 \sin 40^\circ}{\sin 60^\circ}$ (b) $\frac{\sin 60^\circ}{4 \sin 40^\circ}$ (c) $\frac{\sin 40^\circ}{6 \sin 60^\circ}$ (d) $\frac{6 \sin 60^\circ}{\sin 40^\circ}$

44 In ΔABC, if $\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{5}$, then the measure of the biggest angle in the triangle is ...

(a) 60° (b) 75° (c) 90° (d) 120°

45 In ΔXYZ, the expression $\frac{x^2 + y^2 - z^2}{2xy}$

(a) $\cos X$ (b) $\cos Y$ (c) $\cos Z$ (d) $\sin Z$

46) $\lim_{x \to 1} \frac{x^7 - 1}{x - 1} = \ldots$

(a) 7 (b) 8 (c) 6 (d) Zero

47 $\lim_{x \to \infty} \frac{6x}{2x + 3} = \ldots$

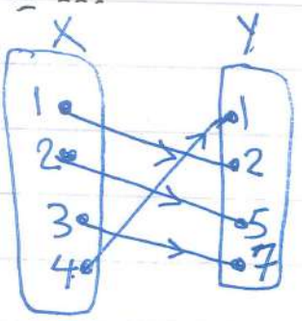
(a) 0 (b) 2 (c) 3 (d) ∞

48 ABC is a triangle in which $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$ then $a : b : c = \ldots$

(a) 6:5:8 (b) 8:5:6 (c) 7:2:4 (d) 3:5:6

49) The opposite figure represents a function $f: X \to Y$, then $f^{-1}(2) = \ldots$

[5 , 1 , 3 , 4]



50) The curve of the function $g: g(x) = x^3 + 4$ is the same curve of $f: f(x) = x^3$ by displacement 4 units in the direction

[$\overrightarrow{oX}$, $\overrightarrow{oX'}$, $\overrightarrow{oY}$, $\overrightarrow{oY'}$]

51) If $f(x) = \sqrt[3]{x}$ and $g(x) = x^3$, then $(f \circ g)(x) = \ldots$

[x^3 , x , $\sqrt[3]{x}$, $x\sqrt[3]{x}$]

52) If $5^x = 2$, then $(25)^x = \ldots$ [10 , 625 , 4 , 2]

53) If f is a function where $f(x) = 7x$, then $f^{-1}(x) = \ldots$

(a) 7 (b) $\frac{x}{7}$ (c) $\frac{7}{x}$ (d) $7 - x$

54) The point of symmetry of the curve of the function $f: f(x) = \frac{1}{x-2}$ is ...

[$(-2,0)$, $(0,2)$, $(2,0)$, $(0,-2)$]

55) The domain of the real function f where $f(x) = \sqrt{x} - 2$ is ...

$[0, \infty[$, $\mathbb{R} - \{0\}$, $[2, \infty[$, $]0, \infty[$

56) If $\log_a 16 = 4$, then $a \in \ldots$

[$\{16\}$, $\{2\}$, $\{2, -2\}$, $\{1\}$]

57 $\lim_{x \to \infty} \frac{x^{-2}+3}{x^{-3}+6} = \ldots$

(a) $\frac{1}{2}$ (b) 2 (c) 3 (d) 6

58 If $2 \sin A = 3 \sin B = 4 \sin C$, then $a : b : c = \ldots$

(a) 2 : 3 : 4 (b) 6 : 4 : 3 (c) 1 : 3 : 2 (d) 5 : 7 : 9

59 If the function f where $f(x) = \begin{cases} \cos 2x + 2, & x \neq 0 \\ a - 1, & x = 0 \end{cases}$

is continuous at $x = 0$, then $a = \ldots$

(a) 1 (b) 2 (c) 3 (d) 4

60 In ΔXYZ, $\frac{x^2+y^2-z^2}{2xy} = \ldots$

(a) $\cos Z$ (b) $\cos X$ (c) $\sin Z$ (d) $\cos Y$

61 $\lim_{x \to \infty} \frac{12^{\frac{1}{x}}}{x} = \ldots$

(a) 0 (b) 1 (c) ∞ (d) 12

62 $\lim_{x \to 0} \frac{x^2 - x + \sin x}{2x} = \ldots$

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

63 In ΔABC, $a : b : c = 3 : 2 : 2$, then $\cos A = \ldots$

(a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

64) From the following functions, the one-to-one function is ... 10

(a) $f_1(x) = x^2$ (b) $f_2(x) = 3x - 1$ (c) $f_3(x) = |x|$ (d) $f_4(x) = 2$

65 The domain of the function $f: f(x) = \frac{5}{x^2 - 9}$ is...

(a) $]-3, 3[$ (b) $\mathbb{R} - [-3, 3]$ (c) $\mathbb{R} -]-3, 3[$ (d) $\mathbb{R} - \{-3, 3\}$

66 If $5^{x-3} = 4^{x-3}$, then $x = \ldots$

(a) 2 (b) 8 (c) 3 (d) 4

67 If the curve $y = \log_4(ax)$ passes through $(1, 2)$, Then $a = \ldots$ (a) 8 (b) 16 (c) 3 (d) 4

68 $f: [-3, 3[\to \mathbb{R}$ where $f(x) = x^2$ is an ... function

(a) odd (b) even (c) otherwise (d) one-to-one

69 If $f(x) = \sqrt{4 - x^2}$, then the domain of $f = \ldots$

(a) $[-2, 2[$ (b) $[-2, 2]$ (c) $]-2, 2[$ (d) $]-2, 2]$

70 If the curve $y = \log_4(1 - ax)$ passes through $(\frac{1}{4}, -\frac{1}{2})$ then $\log_a x = \ldots$ at this point

(a) 2 (b) 3 (c) -2 (d) 4

71 The area included between the curve of the two functions $f: f(x) = |x+3| - 2$, $g: g(x) = 0$ equals ... square units

(a) 2 (b) 3 (c) 4 (d) 5

72) The domain of the function $f: f(x) = \sqrt{4 - x^2}$ is ... (a) $[-2, 2]$ (b) $]-3, 2[$ (c) $[-2, 3[$ (d) $]-3, 3]$

73 If $f(x) = 3x+1$ and $g(x) = x^2 - 1$, then $(f \circ g)(2) = \ldots$

[10 , 3 , 6 , 21]

74 If $\log_3 x = -1$, then $x = \ldots$

[3 , -3 , $\frac{1}{3}$, $\frac{-1}{3}$]

75 If $x^{\frac{3}{2}} = 8$, then $x = \ldots$ [8 , 6 , 4 , 2]

76 If $3^x = 5$, then $x = \ldots$

(a) 3 (b) $\log_3 5$ (c) $\log_5 3$ (d) $\frac{5}{3}$

77 $\log_{0.09} (0.3)^{-2} = \ldots$ [-1 , -2 , $\frac{1}{2}$, $\frac{1}{3}$]

78 If $\log 3 = x$, $\log 4 = y$, then $\log 12 = \ldots$

[$(x+y)$, xy , $x-y$, $\log x + \log y$]

79 If $x = 5 + 2\sqrt{6}$, then $\log\left(x + \frac{1}{x}\right) = \ldots$

[1 , $5 - 2\sqrt{6}$, 10 , $5 + 2\sqrt{6}$]

80 If $a \in \mathbb{R}^+ - \{1\}$, x and $y \in \mathbb{R}^+$, $\log_a y \neq 0$, then

$$\frac{\log_a x}{\log_a y} = \ldots\ldots$$

[$\log_a \frac{x}{y}$, $\log_a (x-y)$, $\log_a x - \log_a y$, $\log_y x$]

81 $\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m} = \ldots$

(a) $\frac{m}{n}$ (b) $\frac{m}{n} a^{m-n}$ (c) $\frac{n}{m} a^{m-n}$ (d) $\frac{n}{m} a^{n-m}$

82 $\lim_{x \to \pi} \frac{\sin x}{\pi - x} = \ldots$

(a) 1 (b) π^2 (c) π (d) $-\pi$

83 If $\lim_{x \to 2} \frac{b}{x+1} = 3$, then $b = \ldots$

(a) 9 (b) 2 (c) 0 (d) 5

84 $\lim_{x \to 0} \frac{\tan^2 2x}{x \sin 3x} = \ldots$

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

85 In ΔABC, if $m(\angle A) = 50^\circ$, $a = 5$ cm. and $b = 6$ cm. Then these are ... solutions.

(a) one (b) two (c) three (d) no

86 $\lim \frac{3x}{\sqrt{9x^2+1}} = \ldots$

(a) $\frac{1}{3}$ (b) zero (c) ∞ (d) 1

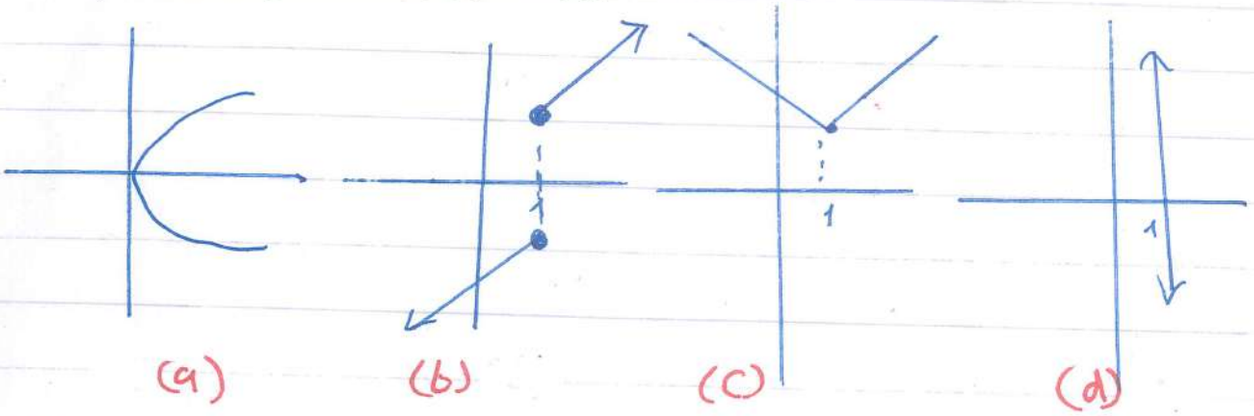
87 In ΔABC, if $a:b:c = 3:2:2$, Then $\cos A = \ldots$

(a) $\frac{1}{8}$ (b) $\frac{-1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

88 $\lim_{n \to \infty} \left(1 + \frac{3}{n}\right) = \ldots$

(a) 4 (b) 3 (c) 1 (d) 2

89) The figure which represents y is fundion in x is



(a) (b) (c) (d)

90) The even fundion from the functions that are defined by the following rules is ...

(a) $f(x) = x^3$ (b) $f(x) = \sin x$

(c) $f(x) = x \cos x$ (d) $f(x) = x \sin x$

91) If f is an even function, $2 \in$ the domain of f, then $f(2) + f(-2) = \ldots$

(a) zero (b) 4 (c) 2 (d) $2f(2)$

92 If f is an odd function, $f(1) = 2$, then which of the following points lies on the curve of f?

(a) $(-1, 2)$ (b) $(-1, -2)$ (c) $(1, -2)$ (d) $(-1, 0)$

93) If $f(x) = 5$, then the domain of the function f is...

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{5\}$ (d) $\mathbb{R} - \{5\}$

94 The domain of the function f: $f(x) = \frac{1}{x-3} + 1$ is ...

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{-3\}$ (d) $\mathbb{R} - \{3\}$

95 The range of the function f: $f(x) = 2 - \frac{3}{x-1}$ is ...

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{3\}$

96 In ΔXYZ, the expression $\frac{x^2+y^2-z^2}{xy} = \ldots$

(a) $\sin Z$ (b) $\cos Z$ (c) $\frac{1}{2}\cos Z$ (d) $2\cos Z$

97 $\lim_{x \to 3} \frac{x-3}{x^2-9} = \ldots$

(a) 3 (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) 6

98 In ΔLMN, if $5\sin L = 3\sin M = 2\sin N$, then $l:m:n = \ldots$

(a) 6:15:10 (b) 6:10:15 (c) 6:5:15 (d) 10:6:15

99 If $a:b:c = 5:8:7$, then $\cos C = \ldots$

(a) $\frac{1}{2}$ (b) 0 (c) -1 (d) 1

100 If $f(x) = \begin{cases} 2x+k, & x > 1 \\ 5-x, & x < 1 \end{cases}$ has a limit at $x = 1$

then $k = \ldots$

(a) -2 (b) 5 (c) 2 (d) 0

101 The length of the radius of the circum circle of the triangle ABC in which $a = 10$ cm. and $m(\angle A) = 30^\circ$ is ... cm

(a) 20 (b) 10 (c) 5 (d) $\frac{1}{5}$

102 $\lim_{x \to 0} \frac{1-\cos x}{\tan 2x} = \ldots$

(a) 1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 0

103 The curve of the function $f: f(x) = 2^{x+2}$ intersects the y-axis at the point ...

[(0,1) , (0,2) , (0,3) , (0,4)]

104 If the function $f: f(x) = (\frac{a}{3})^x$ is an increasing exponential function, then ...

(a) $a > 0$ (b) $a > 1$ (c) $a > 3$ (d) $a < 3$

105 which of the functions that are defined by the following rules represents an exponential growth function?

(a) $f(x) = 2^x$ (b) $f(x) = (\frac{1}{3})^x$

(c) $f(x) = 3^x$ (d) $f(x) = (\frac{2}{3})^x$

106 which of the following functions represents an increasing exponential function on its domain $\mathbb{R}$:

(a) $y = 3(1.05)^x$ (b) $y = 3(\frac{1}{1.05})^x$ (c) $y = 3 + (0.5)^x$

(d) $y = (0.05)^x$

107 An amount of 5000 p. is deposited in a bank gives a yearly compound interest 5% for 7 years = ... pounds

(a) 6750 (b) 7035.5 (c) 5350 (d) 8500

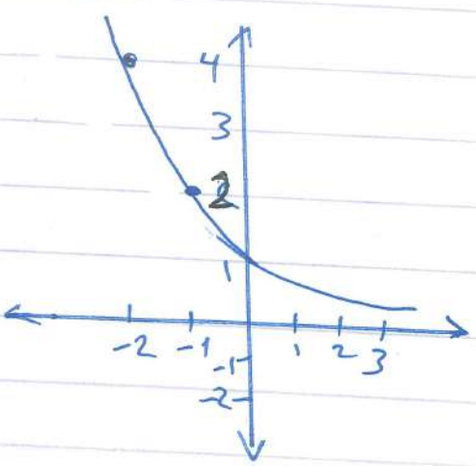
108 The opposite figure shows the function f where ...

(a) $f(x) = 2^{x+1}$

(b) $f(x) = 2^{-x}$

(c) $f(x) = 3^{-x}$

(d) $f(x) = 2^x$



109) $\lim_{x \to \sqrt{2}} \frac{x^5 - 4\sqrt{2}}{x^3 - 2\sqrt{2}} = \ldots$

(a) $\frac{4}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{5}{2}$ (d) $\frac{10}{3}$

110) In ΔABC, if $\frac{\sin A}{3} = \frac{2\sin B}{5} = \frac{\sin C}{4}$, then $a:b:c = \ldots$

(a) 6:5:8 (b) 8:5:6 (c) 7:2:4 (d) 3:5:4

111) $\lim_{x \to 0} \frac{\tan^2 2x}{6x^2} = \ldots$

(a) $\frac{2}{9}$ (b) $\frac{2}{3}$ (c) $\frac{9}{3}$ (d) $\frac{4}{9}$

112) The function $f : f(x) = \begin{cases} kx^2 & , x \leq 2 \\ 2x + k & , x > 2 \end{cases}$

is continuous at $x = 2$, then $k = \ldots$

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{2}{3}$

113) $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 + x - 12} = \ldots$

(a) $\frac{5}{7}$ (b) $\frac{1}{7}$ (c) -1 (d) -5

114) $\lim_{x \to 0} \frac{2x + \sin 3x}{\tan 5x} = \ldots$

(a) 5 (b) $\frac{6}{5}$ (c) 1 (d) Zero

115) If r is the length of the radius of the circumcircle of the triangle XYZ, then $\frac{y}{2\sin y} = \ldots$

(a) $4r$ (b) $2r$ (c) r (d) $\frac{1}{2}r$

116) The number $5^{x+1} + 5^{x}$ is divisible by .... for all natural values of x 17

(a) 7 (b) 6 (c) 13 (d) 17

117) If $2^{x} = 20$, $n < x < n+1$, n is an integer, then $n = \ldots$

(a) 1 (b) 2 (c) 3 (d) 4

118) The two curves of the two functions $f: f(x) = 2^{x}$ and $g: g(x) = 3^{x}$ intersects at $x = \ldots$

(a) -1 (b) 0 (c) 1 (d) 2

119) If f^{-1} is the inverse function of the function f, then....

(a) domain of f^{-1} = domain of f

(b) domain of f^{-1} = range of f

(c) range of f^{-1} = range of f

(d) range of f^{-1} = domain of f^{-1}

120) If the straight line $y = x$ intersects the one-to-one function f in the point $(2, 2)$, then it intersects the function f^{-1} in the point ....

(a) $(-2, 2)$ (b) $(2, 2)$ (c) $(-2, -2)$ (d) $(2, -2)$

121) If the function f^{-1} where $f^{-1} = \{(2, 2), (5, b)\}$ is the inverse of the function f where $f = \{(4, 5), (a, 2)\}$, then $a - b = \ldots$

[zero, 1, -1, 2]

122 In the triangle ABC, $\cos A = \ldots$

(a) $\frac{a^2+b^2-c^2}{2ab}$ (b) $\frac{a^2+c^2-b^2}{2ab}$ (c) $\frac{c^2-a^2-b^2}{2ab}$ (d) $\frac{b^2+c^2-a^2}{2bc}$

123 $\lim_{x \to \infty} \frac{x^3+5}{x(2x^2+3)} = \ldots$

(a) $\frac{5}{8}$ (b) 1 (c) $\frac{5}{3}$ (d) $\frac{1}{2}$

124 The length of the radius of the circumcircle of the triangle ABC in which $m(\angle A) = 30^\circ$ and $a = 10$ cm is ...

(a) 5 cm (b) 10 cm (c) 20 cm (d) 40 cm

125 If the function f where $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ 2a, & x = 1 \end{cases}$ is continuous at $x = 1$, then $a = \ldots$

(a) 2 (b) -2 (c) zero (d) 1

126 The measure of the greatest angle of the triangle whose side lengths are 3, 5, 7 is ...

(a) 150° (b) 120° (c) 60° (d) 30°

127 $\lim_{x \to 0} \frac{2x+3\sin x}{\tan 5x} = \ldots$

(a) 5 (b) $\frac{6}{5}$ (c) 1 (d) zero

128 In ΔABC, $\frac{a}{a+c} = \ldots$

(a) $\frac{\sin A}{\sin B}$ (b) $\frac{\sin A}{\sin C}$ (c) $\frac{\sin A}{\sin A+\sin B}$ (d) $\frac{\sin A}{\sin A+\sin C}$

129 $\lim_{x \to a} \frac{x^9 - a^9}{x - a} = \ldots$

(a) $9a$ (b) $9a^8$ (c) $9a^9$ (d) $9a^{10}$

130 In the triangle ABC, if $a = b$, then $\cos A = \ldots$

(a) $\frac{2b}{c}$ (b) $\frac{c^2}{2b}$ (c) $\frac{c}{2a}$ (d) $\frac{b}{2a}$

131 ABC is a triangle in which $m(\angle A) = 30^\circ$ and $a = 6$ cm, then $\frac{b}{\sin B} = \ldots$

(a) 3 (b) 6 (c) $\frac{1}{5}$ (d) 12

132 $\lim_{x \to \pi} \frac{\sin x}{\pi - x} = \ldots$

(a) 1 (b) -1 (c) π (d) $-\pi$

133 In the triangle XYZ, if $x = y$, then $\cos X = \ldots$

(a) $\frac{2y}{z}$ (b) $\frac{z^2}{2y}$ (c) $\frac{z}{2x}$ (d) $\frac{y}{2x}$

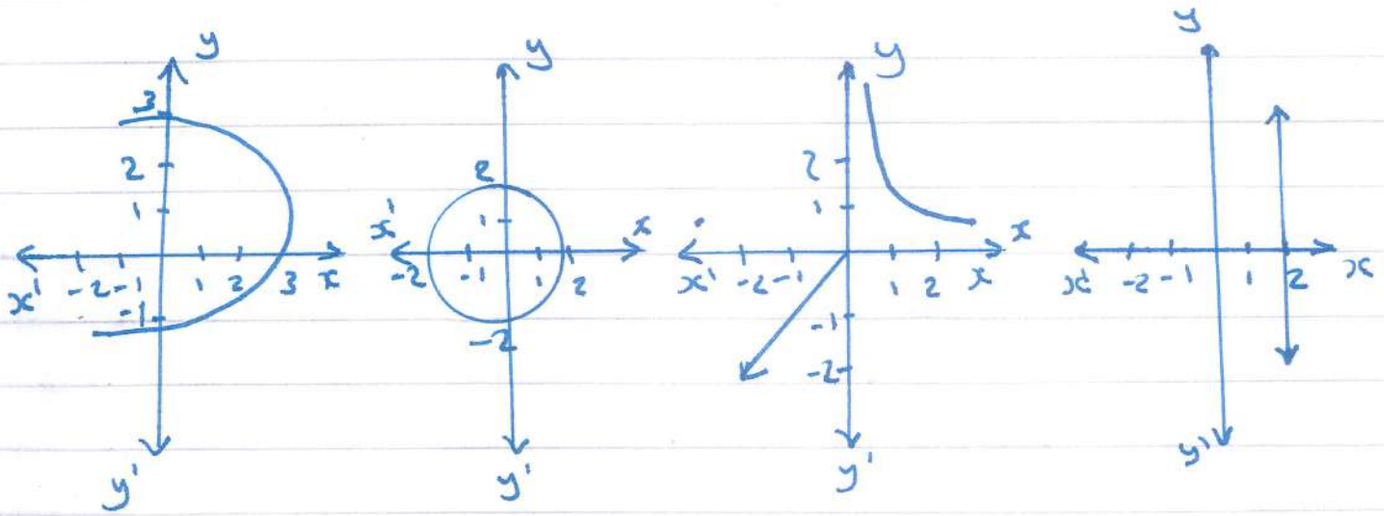
134 $\lim \sqrt{\frac{4}{x} + 1} = \ldots$

(a) 0 (b) 1 (c) 2 (d) ∞

135 If r is the length of the radius of the circumcircle of ΔABC, then $\frac{2b}{\sin B} = \ldots$

(a) r (b) 2r (c) $\frac{1}{2}$ r (d) 4r

136 Which of the following figures represents of function of x ?



(a) (b) (c) (d)

137 $f(x)=\frac{1}{x}$, $g(x)=\sqrt{x}$, then the domain of $(f\cdot g)=$ ...

(a) $\mathbb{R}-\{0\}$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $[0,\infty[$

138 If f is an even function in the interval $[a,b]$ then $b=$ ...

(a) a (b) $-a$ (c) $2a$ (d) a^3

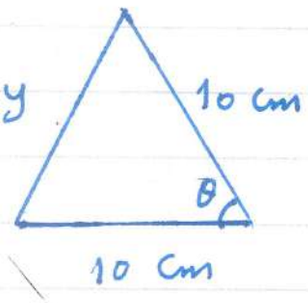
139 The curve of the function $f: f(x)=x^2+4$ is the same of the function $g: g(x)=x^2$ by translation of a magnitude 4 units in direction of ...

(a) $\overrightarrow{ox}$ (b) $\overrightarrow{ox'}$ (c) $\overrightarrow{oy}$ (d) $\overrightarrow{oy'}$

140 The domain of the function $f: f(x)=\frac{5}{\sqrt{x-4}}$ is

(a) $[4,\infty[$ (b) $]4,\infty[$ (c) $]-\infty,4]$ (d) $]-\infty,-4[$

141 In The opposite figure:
At $\theta \to \frac{\pi}{2}$, then: $y \to$ ... cm



(a) zero (b) 5
(c) 10 (d) $10\sqrt{2}$

142 $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \ldots$

(a) -6 (b) zero (c) 3 (d) 6

143 $\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m} = \ldots$

(a) $\frac{m}{n}$ (b) $\frac{m}{n}(a)^{m-n}$
(c) $\frac{n}{m}(a)^{m-n}$ (d) $\frac{n}{m}(a)^{n-m}$

144 $\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9} = \ldots$

(a) $\frac{3}{2}$ (b) $4\frac{1}{2}$ (c) 3 (d) 27

145 $\lim_{x \to \infty} \frac{x^3 + 5}{x(2x^2 + 3)} = \ldots$

(a) $\frac{5}{8}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{5}{3}$

146 $\lim_{x \to 9} \frac{3 - \sqrt{x}}{27 - \sqrt{x^3}} = \ldots$

(a) $\frac{1}{9}$ (b) $\frac{1}{27}$ (c) 3 (d) $-\frac{1}{27}$

147) The symmetric point of the function f: $f(x) = (x-2)^3 + 1$ is ...

(a) (2, 1) (b) (-2, 1) (c) (2, -1) (d) (-2, -1)

148) $f(x) = \frac{1}{x}$, then the symmetric point of the function whose rule $g(x) = f(x+1)$ is ...

(a) (1, 0) (b) (0, 1)

(c) (-1, 0) (d) (-1, 1)

149 The curve of $f: f(x) = |x+3|$ is the same curve of $g: g(x) = |x|$ by translation of magnitude 3 units in direction ...

(a) $\overrightarrow{ox}$ (b) $\overrightarrow{ox'}$

(c) $\overrightarrow{oy}$ (d) $\overrightarrow{oy'}$

150 The domain of the function $f: f(x) = \frac{1}{|x|-3}$ is

(a) $\{3, -3\}$ (b) $[-3, 3]$

(c) $\mathbb{R} - [-3, 3]$ (d) $\mathbb{R} - \{-3, 3\}$

151) The solution set of the equation: $|x-3| + 1 = 0$ in $\mathbb{R}$ is ...

(a) $\mathbb{R}$ (b) $\{-1\}$ (c) $\emptyset$ (d) $\{4\}$

152) The product of the two roots of the equation: $x^2 - 3|x| - 10 = 0$ equals ...

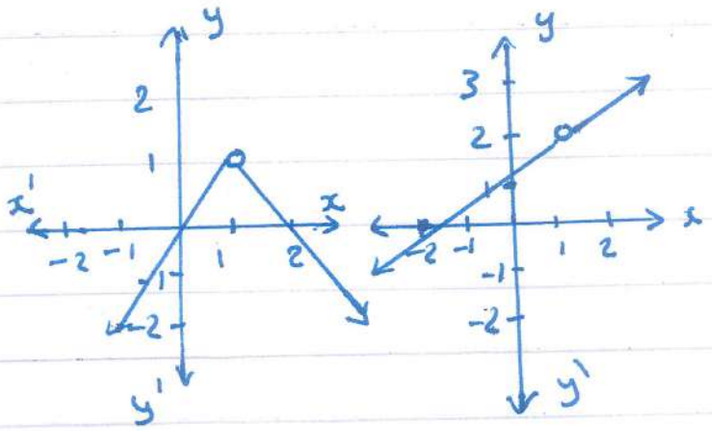
(a) -25 (b) -15 (c) 10 (d) 25

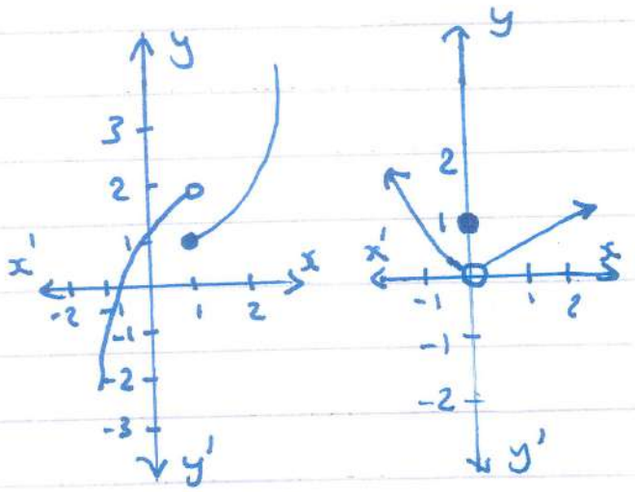
153 If $f(x) = \begin{cases} \frac{x^8 - a^8}{x^5 - a^5} & , x \neq a \\ 200 & , x = a \end{cases}$

is Continuous at $x = a$, then $a = \ldots$

(a) 5 (b) $\frac{8}{5}$ (c) 125 (d) $\frac{1}{5}$

154 which of the following functions has no Limit at $x = 1$?





155 $\lim_{x \to 0} \frac{(x+h)^5 - x^5}{h} = \ldots$

(a) x^5 (b) $5x^4$ (c) zero (d) 1

156 If $n(x)$ is afunction and $\lim_{x \to 2} \frac{n(x) - 8}{x - 2} = 7$,

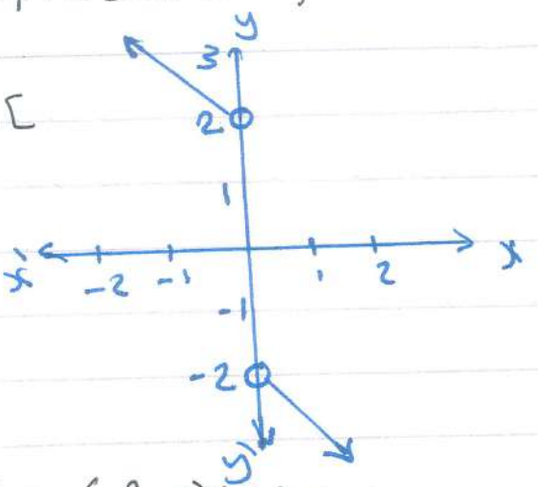
then $\lim_{x \to 2} \frac{2x^2 - n(x)}{x - 2} = \ldots$

(a) 1 (b) 4 (c) 8 (d) 15

157 If f is an odd function in the interval $[a, b]$, then $b = \ldots$

(a) a (b) $-a$ (c) $2a$ (d) a^3

158 The opposite figure represents a function of x whose domain is



(a) $\mathbb{R}$ (b) $\mathbb{R} -]-2, 2[$

(c) $\mathbb{R} - [-2, 2]$ (d) $\mathbb{R} - \{0\}$

159 $f(x) = x + 1$, $g(x) = x^2$, then $(f \circ g)(2) = \ldots$

(a) 3 (b) 4 (c) 5 (d) 9

160 The domain of the function $f: f(x) = \frac{5}{\sqrt[3]{x-8}}$ is

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{8\}$ (d) $[8, \infty[$

161 The function which is one-to-one from the following functions defined by the rules is...

(a) $f_1(x) = x + 2$ (b) $f_2(x) = x^2$

(c) $f_3(x) = |x|$ (d) $f_4(x) = 5$

162 The area bounded between the two curves of the functions $f: f(x) = |x + 3| - 2$, $g(x) =$ zero is ... area units

(a) 2 (b) 3 (c) 4 (d) 5

163 From the following functions, the one-to-one function is ...

(a) $f_1(x) = x + 2$ (b) $f_2(x) = x^2$ (c) $f_3(x) = |x|$

(d) $f_4(x) = 5$

164 If $f(x) = 3x + 1$, $g(x) = x^3$, then $(g \circ f)(x) = \ldots$

(a) $3x^4 + x^3$ (b) $(3x+1)^3$ (c) $3x^3 + 1$ (d) $x^3 + 3x + 1$

165 The inverse function of $f(x) = x + 2$ is $f^{-1}(x) = \ldots$

(a) $x + 2$ (b) $-x + 2$ (c) $x - 2$ (d) $\frac{x}{2}$

166 If $\log_x 4 = 2$, then $x = \ldots$

(a) 4 (b) ± 2 (c) 2 (d) -2

167 If $f(x) = x + 1$ and $g(x) = x^2$, then $(f \circ g)(2) = \ldots$

(a) 3 (b) 4 (c) 9 (d) 5

168 If $\log_x 4 = 2$, then $x = \ldots$

(a) 4 (b) ± 2 (c) 2 (d) -2

169 If $5^{x-1} = 3^{1-x}$, then $x = \ldots$

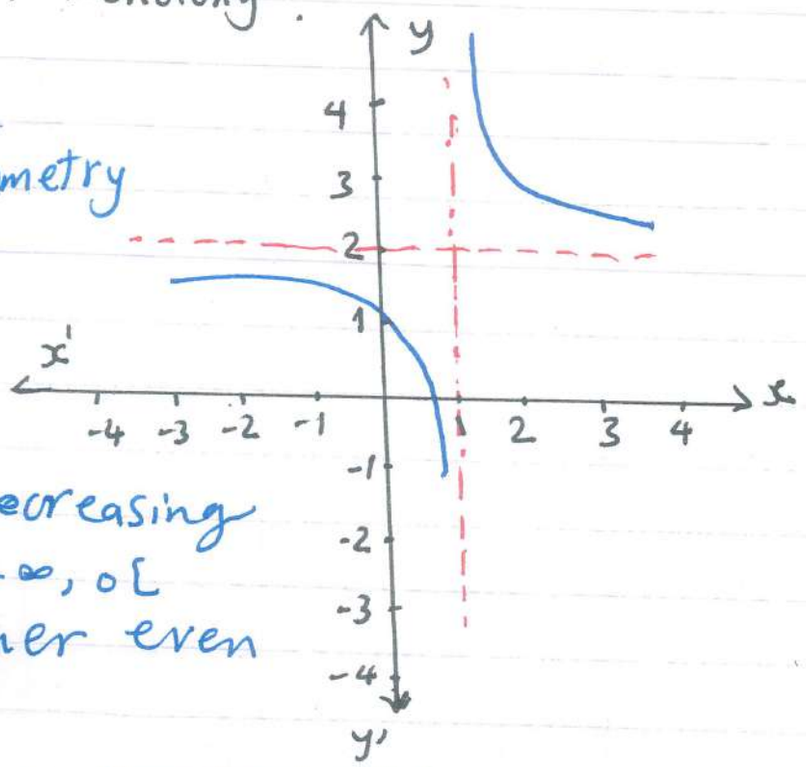
(a) 1 (b) 2 (c) 3 (d) 5

170 If f is an odd function on $[-x, x]$, then $f(-x) + f(x) = \ldots$

(a) $2x$ (b) $-2x$ (c) 0 (d) undefined

171 Graph the function f where $f(x)=\frac{1}{x-1}+2$ then from the graph determine its range and deduce it monotony. 27)

Solution



the point of symmetry is $(1,2)$

Range $= \mathbb{R}-\{2\}$

The function is decreasing on $]0,\infty[$, $]-\infty,0[$

The function neither even nor odd.

172 Find the domain of f & g:

① $f(x)=\frac{x}{\sqrt{1-x}}$ ② $g(x)=\frac{x-1}{x^2-x}+\frac{1}{x+1}$

Solution:

① $1-x>0 \Rightarrow -x>-1 \Rightarrow x<1$

Domain $=]-\infty, 1[$

②

$x^2-x=0 \Rightarrow x(x-1)=0 \Rightarrow x=0,\ x=-1$

, $x+1=0 \Rightarrow x=-1$

Domain $= \mathbb{R}-\{0,1,-1\}$

173 Draw the graph of the functions f, g :

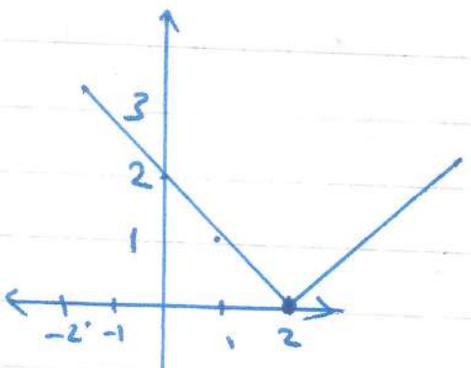
① $f(x) = \sqrt{x^2-4x+4}$, $g(x) = |x^2-4x+5|$, $x \in [0,4]$

then deduce its Range and discuss its monotonicity

Solution :

① $f(x) = \sqrt{(x-2)^2} = |x-2|$

point of symmetry $(2,0)$



Range = $[0, \infty[$

the function is decreasing on $]-\infty, 2[$

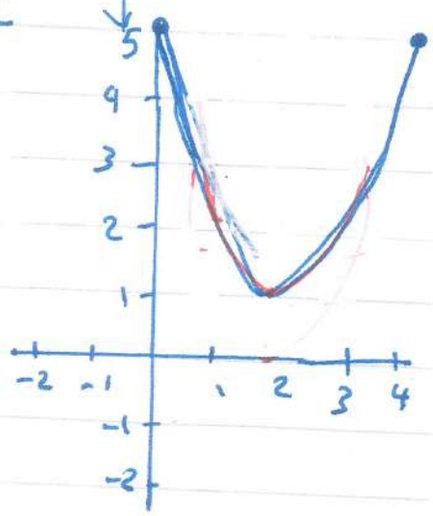
, increasing on $]2, \infty[$

② $g(x) = |x^2-4x+4+1|$

$= |(x-2)^2+1|$

$= (x-2)^2+1$

point of symmetry is $(2,1)$



Range = $[1, 5[$

the function is decreasing on $]-\infty, 2[$

, increasing on $]2, 4[$

174 Find: $\lim_{x\to 0} \frac{x^2+\sin 3x}{5x\cos 2x}$

Solution:

$$\lim_{x\to 0} \frac{x^2+\sin 3x}{5x\cos 2x} = \lim_{x\to 0} \frac{x\left(x+\frac{\sin 3x}{x}\right)}{x(5\cos 2x)}$$

$$= \lim_{x\to 0} \frac{x+\frac{\sin 3x}{x}}{5\cos 2x} = \frac{0+3}{5} = \frac{3}{5}$$

175 Solve the triangle ABC in which $a=9$ cm, $b=15$ cm, $m(\angle C)=106^\circ$

Solution:

$a=9$ cm, $b=15$ cm, $m(\angle C)=106^\circ$

$$c^2 = a^2+b^2-2ba\cos C$$
$$= 81+225-2\times 9\times 15\cos 106 \simeq 380.42$$
$$\Rightarrow c = 19.5 \text{ cm}$$

$$\sin A = \frac{b^2+c^2-a^2}{2bc} = \frac{(15)^2+(19.5)^2-(9)^2}{2\times 15\times 19.5} \simeq 0.896$$

$$\Rightarrow m(\angle A) = 26^\circ\ 18'\ 17.88''$$
$$m(\angle B) = 108-(26^\circ\ 18'\ 17.88''+106) = 47^\circ\ 41'\ 42.12''$$

176 Find: $\lim_{x\to -2} \frac{(x+3)^5-1}{x^2-4}$

$$= \lim_{x+3\to +1} \frac{(x+3)^5-1}{(x-2)(x+2)} = \lim_{x+3\to 1} \frac{(x+3)^5-1}{(x-2)((x+3)-1)}$$

$$= \frac{1}{-4}\times\frac{5}{1}(1)^{5-1} = \frac{-1}{4}\times 5 = -\frac{5}{4}$$

[17] Tell whether each of the functions defined by the following rules is odd, even or otherwise.

① $f_1(x) = x \cos x$ $\quad f_2(x) = \begin{cases} x^2, & x \geq 0 \\ |x|, & x < 0 \end{cases}$

③ $f_3(x) = x^2|x| - 1$

Solution:

① $f_1(-x) = (-x)\cos(-x)$

$= -x\cos x = -f_1(x)$

$\therefore f_1(x)$ is odd function.

② $f_2(-x) = \begin{cases} (-x)^2, & -x \geq 0 \\ |-x|, & -x < 0 \end{cases}$

$= \begin{cases} x^2, & x \leq 0 \\ |x|, & x > 0 \end{cases}$

$\neq f_2(x) \neq f_2(-x)$

$f_2(x)$ ~~Neither~~ even nor odd.

③ $f_3(-x) = (-x)^2|-x| - 1$

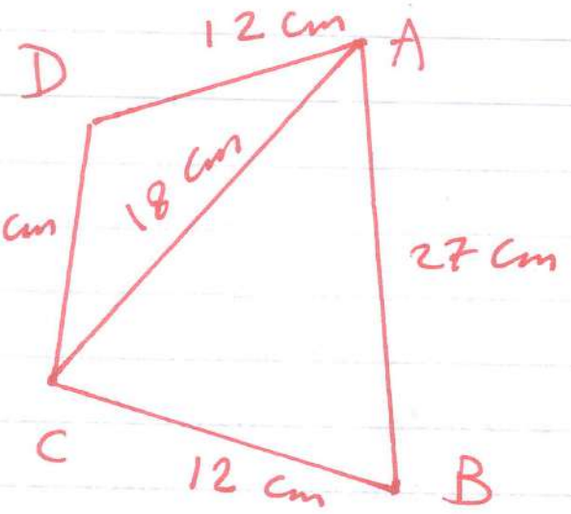
$= x^2|x| - 1$

$= f(x)$

$f(x)$ is even function.

(178) ABCD is a quadrilateral in which AB = 27 Cm., BC = 12 Cm., CD = 8 Cms., DA = 12 Cm., AC = 18 Cm. prove that $\overrightarrow{AC}$ bisects $\angle BAD$, then find the area of the shape ABCD (31)

Solution:



In ΔABC

$$\cos(B\hat{A}C) = \frac{(27)^2 + (18)^2 - (12)^2}{2 \times 27 \times 18}$$

$$m(\angle BAC) = 20^\circ\ 44'\ 30.9''$$

In ΔADC

$$\cos(\angle DAC) = \frac{(12)^2 + (18)^2 - (8)^2}{2 \times 12 \times 18}$$

$$\Rightarrow m(\angle DAC) = 20^\circ\ 44'\ 30.9''$$

$$\Rightarrow \therefore m(\angle DAC) = m(\angle BAC)$$

$\therefore \overrightarrow{AC}$ bisects $\angle BAD$ (first)

the area of ABCD = the area of ΔABC + the area of ΔADC

$$= \tfrac{1}{2} \times 27 \times 18 \times \sin 20^\circ 44' 30.9'' + \tfrac{1}{2} \times 18 \times 12 \sin 20^\circ 44' 30.9''$$

$$\simeq 124\ Cm^2$$

179 Find in $\mathbb{R}$ s.s of each of the following 32

① $|x| + x = 0$

② $|2x-3| - |6-4x| > 0$

Solution:

①

$x \geq 0$	$x < 0$
$x + x = 0$	$-x + x = 0$
$2x = 0$	$0 = 0$
$x = 0$	$\forall x \in]-\infty, 0[$

$S.S =]-\infty, 0]$

② $|2x-3| - |4x-6| > 0$

$\Rightarrow |2x-3| - 2|x-3| > 0$

$-|2x-3| > 0$

$\Rightarrow |2x-3| < 0$ Refused

$S.S = \phi$

180 without using Calculator find the value of:

$$\text{Log } 25 + \frac{\text{Log } 8 \times \text{Log } 16}{\text{Log } 64}$$

Solution:

$$= \text{Log } 25 + \frac{\text{Log } 2^3 \times \text{Log } 2^4}{\text{Log } 2^6}$$

$$= \text{Log } 25 + \frac{3 \log 2 \times 4 \log 2}{6 \log 2} = \text{Log } 25 + 2 \log 2$$

$$= \text{Log } 25 + \text{Log } 4$$

$$= \text{Log } 100 = 2$$

181 Find (1) $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x^2+x}$ (2) $\lim_{x\to\infty} \frac{1}{x}\sqrt{3+4x^2}$

Solution:

(1) $$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x^2+x} \times \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$$

$$= \lim_{x\to 0} \frac{(x+4-4)}{(x^2+x)(\sqrt{x+4}+2)}$$

$$= \lim_{x\to 0} \frac{x}{x(x+1)(\sqrt{x+4}+2)}$$

$$= \lim_{x\to 0} \frac{1}{(x+1)(\sqrt{x+4}+2)} = \frac{1}{4}$$

(2)

$$\lim_{x\to\infty} \frac{1}{x}\sqrt{3+4x^2} = \lim_{x\to\infty} \sqrt{\frac{3}{x^2}+\frac{4x^2}{x^2}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{3}{x^2}+4}$$

$$= \sqrt{0+4} = 2$$

182 If: $f(x) = x^2 - 1$, $g(x) = x + 1$ 34

graph the Curve of the function $\frac{f}{g}$,

From the graph, deduce the domain and the range, then investigate its monotony.

Solution:

the Domain of f is $\mathbb{R}$

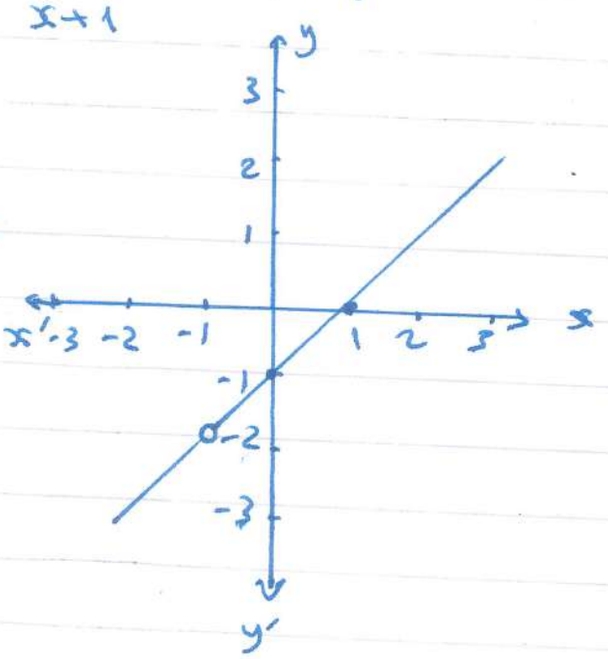
the ~~Range~~ of g is $\mathbb{R}$

the Domain of $\frac{f}{g}$ is $\mathbb{R} - \{-1\}$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2-1}{x+1} = \frac{(x-1)(x+1)}{x+1} = x - 1$$

the rang $= \mathbb{R} - \{-2\}$

The function is increasing on $\mathbb{R} - \{-1\}$

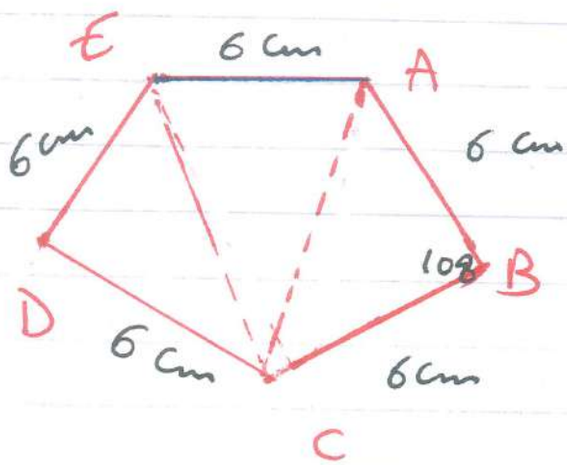


183

35

If the perimeter of a regular pentagon is 30 cm. find its surface area.

Solution



In ΔABC

from cosin rule

$$(AC)^2 = (AB)^2 + (BC)^2 - 2\,AB \times BC \cos(\angle ABC)$$

$$= 6^2 + 6^2 - 2 \times 6 \times 6 \cos 108 \doteq 94.25$$

$$\Rightarrow AC = 9.7 \text{ cm}$$

Area of the pentagon =

2 Area of (ΔABC) + Area of $\Delta(ACE)$

$$2 \times \tfrac{1}{2} \times 6 \times 6 \sin 108 + \tfrac{1}{2} \times 9.7 \times 9.7 \times \sin 36$$

$$\simeq 62 \text{ cm}^2$$

184 If the function f where $f(x)=\frac{1}{x}$, find the range of the function f, the two coordinates of the symmetry point of the curve, then find in $\mathbb{R}$ the solution set of the equation $f(\frac{1}{x})=4$ 36

Solution:

Range $= \mathbb{R}-\{0\}$ the two coordinates are $x=0$, $y=0$

$f(\frac{1}{x})=4$

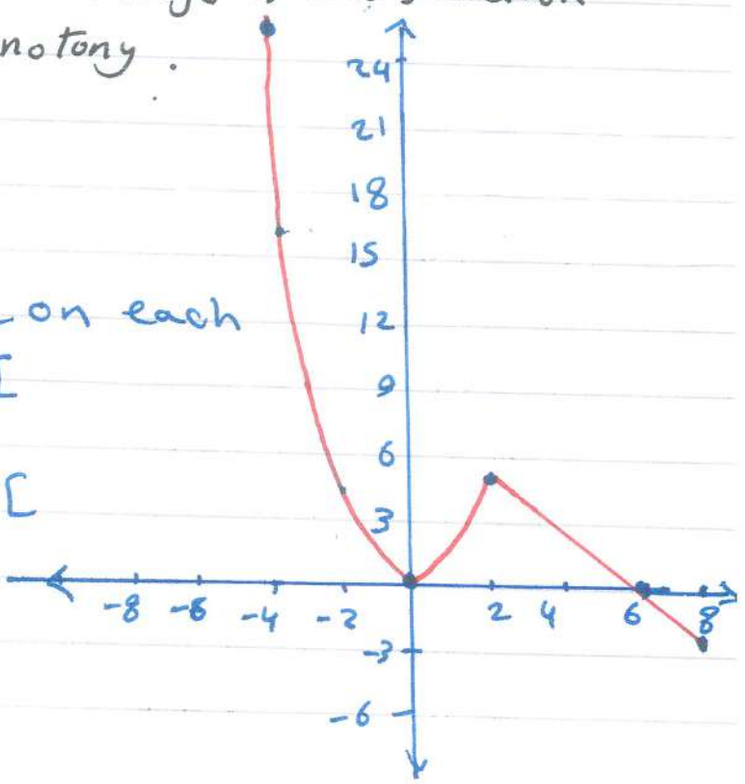
$\Rightarrow x=4$ $\quad S.S=\{4\}$

185 graph the curve of the function f where

$$f(x)=\begin{cases} x^2 & \text{when } -5\le x<2 \\ 6-x & \text{when } 2\le x\le 8 \end{cases}$$

From the graph, determine the range of the function and Investigate its monotony.

Solution:



range $=[-2, 25]$

the function is decreasing on each of $]-5, 0[$, $]2, 8[$

; increasing on $]0, 2[$

186 Find :

(1) $\lim_{x \to \infty} \frac{4-3x^2}{\sqrt{x^4+5}}$ (2) $\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3}$

Solution :

(1) $\lim_{x \to \infty} \frac{4-3x^2}{\sqrt{x^4+5}} = \lim_{x \to \infty} \frac{\frac{4}{x^2} - \frac{3x^2}{x^2}}{\sqrt{\frac{x^4}{x^4} + \frac{5}{x^4}}} = \frac{0-3}{\sqrt{1+0}} = \frac{3}{1} = 3$

(2) $\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3}$

$= \lim_{x+1 \to 4} \frac{(x+1)^{\frac{1}{2}} - 4^{\frac{1}{2}}}{(x+1)-4} = \frac{1}{2}(4)^{\frac{1}{2}-1} = \frac{1}{4}$

187 Find the perimeter of ΔABC in which $a = 8$ Cm., $b = 6$ Cm. , $m(\angle C) = 48^\circ$

Solution :

$c^2 = a^2 + b^2 - 2ab \cos C$

$= 8^2 + 6^2 - 2 \times 8 \times 6 \cos 48 = 35.76$

$\Rightarrow c = 5.98$ Cm

perimeter of $\Delta ABC = a + b + c = 6 + 8 + 5.98$

$\simeq 19.98$ Cm

188 Find :

(1) $\lim_{x \to \infty} \frac{5x^4 + 3x^2 - 6}{2x + x^4}$ (2) $\lim_{x \to -2} \frac{x+2}{x-3}$

Solution:

(1) $$\lim_{x \to \infty} \frac{\frac{5x^4}{x^4} + \frac{3x^2}{x^4} - \frac{6}{x^4}}{\frac{2x}{x^4} + \frac{x^4}{x^4}} = \frac{5 + 0 - 0}{0 + 1} = \frac{5}{1} = 5$$

(2) $$\lim_{x \to -2} \frac{x+2}{x-3} = \frac{-2+2}{-2-3} = \frac{0}{-5} = 0$$

---

(189) If ABC is a triangle in which

$\frac{1}{2}\sin A = \frac{1}{3}\sin B = \frac{1}{4}\sin C$, find the measure of its largest angle.

Solution:

$$\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{4}$$

$a = 2k$, $b = 3k$, $c = 4k$

the largest angle is C

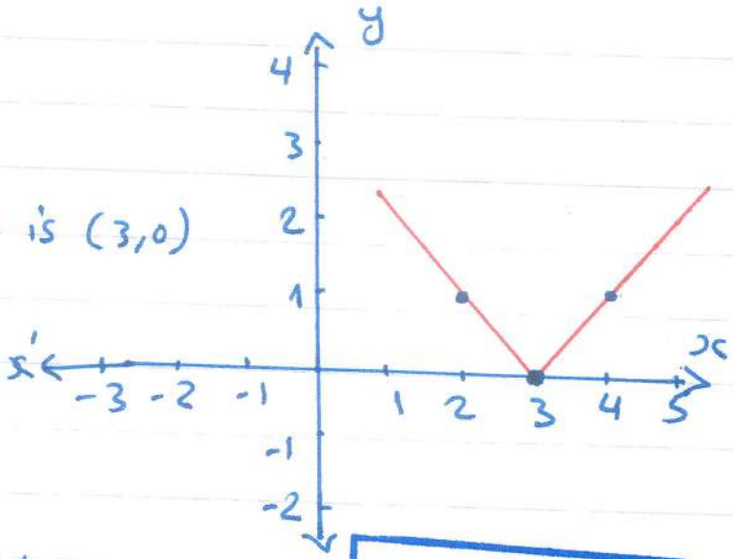
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{4k^2 + 9k^2 - 16k^2}{2 \times 2k \times 3k}$$

$$= \frac{-3k^2}{12k^2} = -\frac{1}{4}$$

$$\Rightarrow m(\angle C) = 104^\circ\ 28'\ 39''$$

18) Graph the curve of the function f where $f(x) = |x-3|$, deduce the range and the monotony of the function and tell whether it is even, odd or otherwise. 39

Solution:



The point of symmetry is (3,0)

the range $= [0, \infty[$

the function f is
decreasing on $]-\infty, 3[$,
increasing on $]3, \infty[$

the function niether even nor odd

---

19) Find the solution set for each of the following in $\mathbb{R}$:

① $|x-3| \geq 5$ ② $|x-3| = 0$

Solution:

① $|x-3| \geq 5$

$x \geq 3$ | $x < 3$

$x-3 \geq 5$; $x-3 \leq -5$

$x \geq 8$ | $x \leq -2$

$S.S = \mathbb{R} -]-2, 8[$

② $|x-3| = 0$

$x-3 = 0$

$x = 3$

$S.S = \{3\}$

192) Find the solution set for each of the following in $\mathbb{R}$:

① $\log x = \log 3 + \log 10$ ② $9^x - 3 \times 3^x = 0$

Solution:

① $\log x = \log (3 \times 10)$

$\log x = \log 30$

$\Rightarrow x = 30$ $\quad S.S = \{30\}$

② $9^x - 3 \times 3^x = 0$

$\Rightarrow 3^{2x} - 3 \times 3^x = 0$

$3^x (3^x - 3) = 0$

$\Rightarrow 3^x = 0$ or $3^x - 3 = 0$

x is undefined $\quad 3^x = 3$

$x = 1$

$S.S = \{1\}$

193) without using calculator, find in the simplest form the value of: $\frac{1}{\log_2 30} + \frac{1}{\log_3 30} + \frac{1}{\log_5 30}$

Solution:

$$\text{expression} = \frac{\log 2}{\log 30} + \frac{\log 3}{\log 30} + \frac{\log 5}{\log 30}$$

$$= \frac{\log 2 + \log 3 + \log 5}{\log 30} = \frac{\log (2 \times 3 \times 5)}{\log 30} = \frac{\log 30}{\log 30} = 1$$

194 Find

(1) $\lim_{x \to 3} \frac{x^2 - 6x + 9}{x - 3}$ (2) $\lim_{x \to 2} \frac{2x^2 - 8}{x - 2}$

Solution:

① $\lim_{x \to 3} \frac{(x-3)^2}{(x-3)} = \lim_{x \to 3} (x-3) = 0$

② $\lim_{x \to 2} \frac{2(x^2-4)}{x-2} = \lim_{x \to 2} \frac{2(x-2)(x+2)}{(x-2)}$

$= \lim_{x \to 2} 2(x+2) = 8$

195) Find the diameter Length of the circumcircle of ΔABC in each of the following cases:

(1) $m(\angle A) = 75°$, $a = 21$ cm

(2) $m(\angle B) = 50°$, $m(\angle C) = 65°$, $c - b = 6$ cm

Solution:

①

$$\frac{a}{\sin A} = 2r$$

$$\Rightarrow d = 2r = \frac{21}{\sin 75} \simeq 21.7 \text{ cm}$$

②

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$

$$\Rightarrow \frac{c - b}{\sin C - \sin B} = 2r$$

$$\Rightarrow d = 2r = \frac{6}{\sin 65 - \sin 50} = 42.8 \text{ cm}$$

196 Reduce:

(1) $\frac{4^{2n+1} \times 2^{1-n}}{8^{n+2}}$ (2) $\log_6 54 - \log_6 9$

Solution:

(1) $\frac{(2^2)^{2n+1} \times 2^{1-n}}{(2^3)^{n+2}} = \frac{2^{4n+2} \times 2^{1-n}}{2^{3n+6}}$

$= 2^{4n+2+1-n-3n-6}$

$= 2^{-3} = \frac{1}{8}$

(2) $\log_6 54 - \log_6 9$

$= \log_6 \frac{54}{9} = \log_6 6 = 1$

197 tell whether each of the functions defined by the following rules is odd or even:

(1) $f(x) = x + \sin x$ (2) $f(x) = x^3 - 2x^2$

Solution:

(1) $f(-x) = (-x) + \sin(-x)$

$= -x - \sin x$

$= -(x + \sin x) = -f(x)$ f is odd function

(2) $f(-x) = (-x)^3 - 2(-x)^2$

$= -x^3 - 2x^2 \neq f(x)$

$-(x^3 + 2x^2) \neq -f(x)$

the function niether even nor odd

198) Find the value of the following: 43

(1) $\lim_{x \to 3} \frac{(x-6)^2 - 9}{x^2 - 9}$ (2) $\lim_{x \to -1} \frac{2x^3 - x^2 - 2x + 1}{x^3 + 1}$

Solution

(1) $\lim_{x \to 3} \frac{(x-6+3)(x-6-3)}{(x-3)(x+3)}$

$= \lim_{x \to 3} \frac{(x-3)(x-9)}{(x-3)(x+3)} = \lim_{x \to 3} \frac{x-9}{x+3} = \frac{3-9}{3+3} = \frac{-6}{6} = -1$

(2) $\lim_{x \to -1} \frac{(x+1)(2x^2 - 3x + 1)}{(x+1)(x^2 - x + 1)}$

$= \lim_{x \to -1} \frac{2x^2 - 3x + 1}{x^2 - x + 1}$

$= \frac{2(-1)^2 - 3(-1) + 1}{(-1)^2 - (-1) + 1} = \frac{6}{3} = 2$

$x+1 \;|\; 2x^3 - x^2 - 2x + 1$

$2x^2 - 3x + 1$

$2x^3 + 2x^2$

$-3x^2 - 2x + 1$

$-3x^2 - 3x$

$x + 1$

$x + 1$

0

199) ABC is a triangle in which $m(\angle A) = 36°$, $m(\angle C) = 45°$ and $b = 9$ cm, Find the area of the circumcircle of the triangle.

Solution:

$m(\angle B) = 180 - (36 + 45) = 99°$

$2r = \frac{b}{\sin B} = \frac{9}{\sin 99} = 9.11 \Rightarrow r = 4.56$ cm

area of circle $= \pi r^2 = 3.14 \times (4.56)^2 \simeq 65.2$ cm^2

200 If $f(x) = |x-3| + |x+2|$, prove that: $f(2) = f(-1)$

Solution:

$$f(2) = |2-3| + |2+2| = 1+4 = 5$$

$$f(-1) = |-1-3| + |-1+2| = 4+1 = 5$$

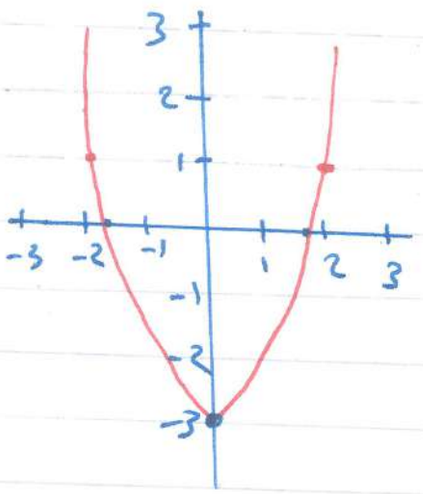
$\therefore f(2) = f(-1)$

201 Use the curve of the function f where $f(x) = x^2$ to graph the following functions:

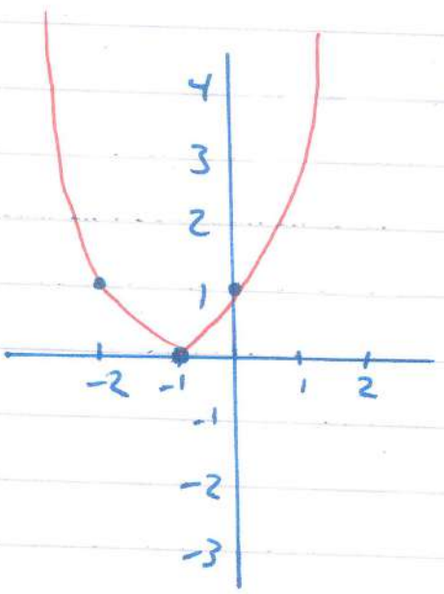
(1) $f_1 : f_1(x) = x^2 - 3$ (2) $f_2 : f_2(x) = (x+1)^2$

Solution:

(1) the point of symmetry of f_1 is $(0, -3)$



(2) the point of symmetry of f_2 is $(-1, 0)$



202 Find :

(1) $\lim_{x \to 2} \frac{x^5 - 32}{x - 2}$ (2) $\lim_{x \to 1} \frac{(x-2)^4 - 1}{x - 1}$

Solution :

(1) $\lim_{x \to 2} \frac{x^5 - 2^5}{x - 2} = \frac{5}{1}(2)^{5-1} = 80$

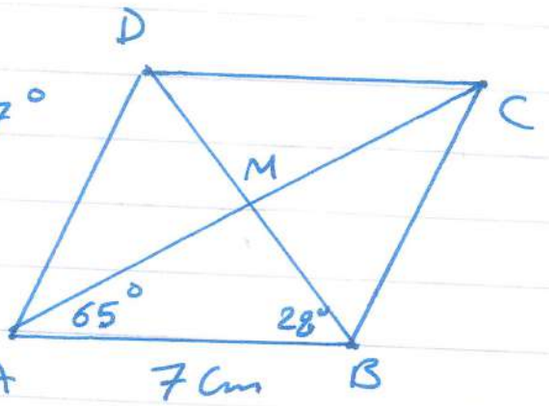
(2) $\lim_{x \to 1} \frac{(x-2)^4 - (-1)^4}{(x-2) + 1}$

$= \lim_{x-2 \to -1} \frac{(x-2)^4 - (-1)^4}{(x-2) - (-1)} = \frac{4}{1}(-1)^{4-1} = -4$

203 ABCD is a parallelogram in which AB = 7 cm, the two diagonals $\overline{AC}$ and $\overline{BD}$ form two angles of measurements 65° and 28° with $\overline{AB}$ respectively, find the lengths of $\overline{BD}$ and $\overline{AC}$

solution :

$m(\angle AMB) = 180 - (28^\circ + 65^\circ) = 87^\circ$



from sine rule :

$\frac{MB}{\sin 65} = \frac{MA}{\sin 28} = \frac{7}{\sin 87}$

$MB = \frac{7 \sin 65}{\sin 87} = 6.35 \Rightarrow DB = 2MB = 12.7$ Cm

$MA = \frac{7 \sin 28}{\sin 87} = 3.29 \Rightarrow AC = 2AM = 6.58$ Cm

204 Find the solution set of each of the following equations in $\mathbb{R}$:

① $\log_2 x + \log_2 (x+1) = 1$ ② $3^x + 3^{1+x} = 36$

Solution:

① $\log_2 x(x+1) = 1$

$\Rightarrow x(x+1) = 2^1$

$\Rightarrow x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

$x = 1$ or $x = -2$ refused

$S.S = \{1\}$

② $3^x + 3^{1+x} = 36$

$3^x (1+3) = 36$

$4 \times 3^x = 36 \quad \div 4$

$3^x = 9$

$3^x = 3^2$

$x = 2$

$S.S = \{2\}$

205) Find:

(1) $\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$ (2) $\lim_{x \to \infty} \frac{4x^2 + 1}{x^2 - 2}$

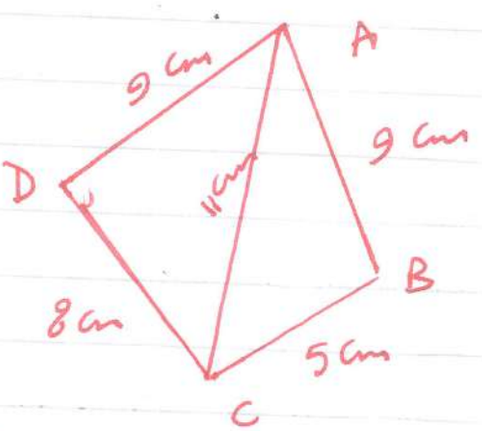
Solution:

(1) $\lim_{x \to 3} \frac{x^3 - 3^3}{x^2 - 3^2} = \frac{3}{2}(3)^{3-2} = \frac{9}{2}$

(2) $\lim_{x \to \infty} \frac{\frac{4x^2}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x^2}} = \frac{4+0}{1-0} = \frac{4}{1} = 4$

206 ABCD is a quadrilateral in which AB = 9 cm., BC = 5 cm. CD = 8 cm., DA = 9 cm. and AC = 11 cm prove that ABCD is asyclic quadrilateral.

Solution:



In Δ ABC

$\cos(\angle B) = \frac{5^2 + 9^2 - 11^2}{2 \times 5 \times 9} = -\frac{1}{6}$

$\Rightarrow m(\angle B) = 99^\circ\ 35'\ 38.65''$

In Δ ADC

$\cos(D) = \frac{9^2 + 8^2 - 11^2}{2 \times 9 \times 8} = \frac{1}{6}$

$m(\angle D) = 80^\circ\ 24'\ 21.35''$

$m(\angle B) + m(\angle D) = 99^\circ\ 35'\ 38.65 + 80^\circ\ 24'\ 21.35 = 180$

$\therefore$ ABCD is asyclic quadri lateral

20-7 Find the solution set of the following equation in $\mathbb{R}$: $4^x + 2^{x+1} = 8$ 48

Solution:

$$2^{2x} + 2^{x+1} - 8 = 0$$

$$2^{2x} + 2 \times 2^{x} - 8 = 0$$

$$(2^x - 2)(2^x + 4)$$

$2^x = 2^1$	$2^x = -4$
$x = 1$	Refused
$S.S = \{1\}$	

20-8 Without using the calculator, prove that:

$$\log_6 8 + \log_6 27 = \log_3 27$$

Solution:

$$L.H.S = \log_6 8 \times 27 = \log_6 216 = \log_6 6^3$$

$$= 3 \log_6 6 = 3 \quad \ldots (1)$$

$$R.H.S = \log_3 3^3 = 3 \log_3 3 = 3 \quad \ldots (2)$$

from 1, 2

$$R.H.S = L.H.S$$

209 Find

① $\lim_{x \to 1} \frac{x^2+5x-6}{x^2-1}$ ② $\lim_{x \to 1} \frac{(x+1)^5-32}{x-1}$

Solution:

① $\lim_{x \to 1} \frac{(x-1)(x+6)}{(x-1)(x+1)} = \lim_{x \to 1} \frac{x+6}{x+1} = \frac{1+6}{1+1} = \frac{7}{2}$

② $\lim_{x+1 \to 2} \frac{(x+1)^5 - 2^5}{(x+1) - 2} = \frac{5}{1}(2)^{5-1} = 80$

210 ABC is a triangle in which $\cos A = \frac{2}{5}$, $b = 2\frac{1}{2}$ cm. and $c = 2$ cm. prove that the triangle is isosceles.

Solution

$a^2 = b^2 + c^2 - 2bc \cos A$

$= \frac{25}{4} + 4 - 2 \times \frac{5}{2} \times 2 \times \frac{2}{5} = \frac{25}{4}$

$\Rightarrow a = \frac{5}{2} = 2\frac{1}{2}$ cm

$\therefore a = b$

$\therefore$ the triangle is isosceles triangle

211 Find the solution set of the inequality:

$|x|+1 < 2$ in $\mathbb{R}$

Solution:

$|x|+1 < 2$

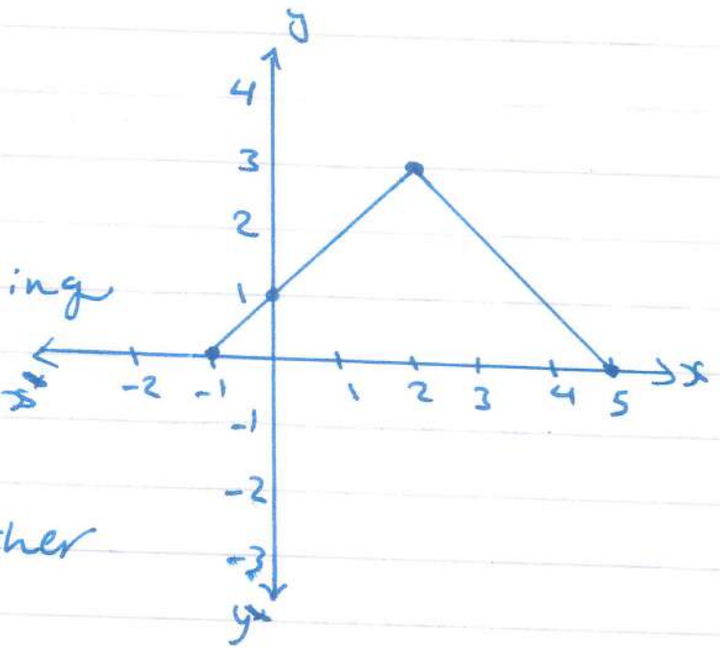
$|x| < 1$

$-1 < x < 1$ $\quad S.S =]-1, 1[$

212 Graph the function f where $f(x) = \begin{cases} x+1, & -1 \le x < 2 \\ 5-x, & 2 \le x \le 5 \end{cases}$

From the graph, deduce the range of this function, investigate it monotony and tell whether it is even, odd or otherwise.

Solution:



Range = $[0, 3]$

the function is decreasing on $]2, 5[$,
increasing on $]-1, 2[$

the function is niether even nor odd

213 Find

(1) $\lim_{x \to 1} \frac{x^3 - 2x + 1}{x^2 - 1}$ (2) $\lim_{x \to 1} \left(\frac{1}{x} + 3\right)$

Solution:

① $\lim_{x \to 1} \frac{(x-1)(x^2+x-1)}{(x-1)(x+1)} = \lim_{x \to 1} \frac{x^2+x-1}{x+1} = \frac{1}{2}$

② $\lim_{x \to 1} \left(\frac{1}{x} + 3\right) = \frac{1}{1} + 3 = 1 + 3 = 4$

214 ABC is a triangle in which $m(\angle B) = 35°$, $m(\angle C) = 70°$, and the radius length of the circumcircle of the Triangle = 16 Cm., find the area and perimeter of triangle ABC to the nearest integer.

Solution:

$m(\angle A) = 180° - (35 + 70) = 75°$

from sine rule:

$$\frac{a}{\sin 75} = \frac{b}{\sin 35} = \frac{c}{\sin 70} = 32$$

$a = 32 \sin 75° \simeq 30.91$ Cm

$b = 32 \sin 35° \simeq 18.35$ Cm

$c = 32 \sin 70 \simeq 30.1$ Cm

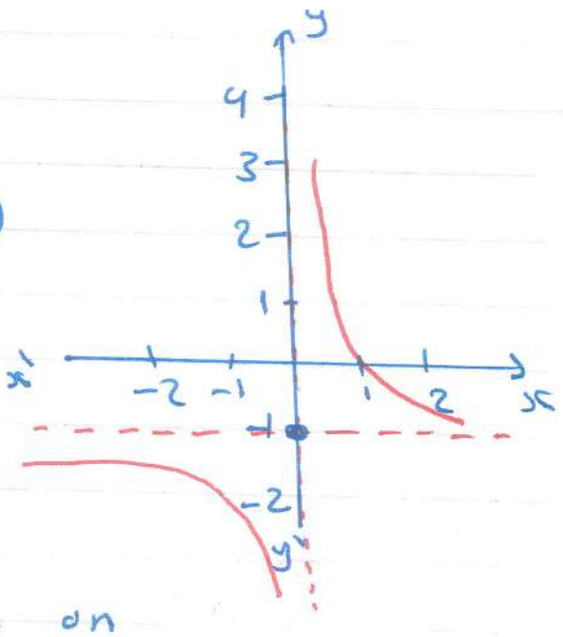
Area of triangle ABC $= \frac{1}{2} \times 30.91 \times 18.35 \sin 70°$

$\simeq 267$ Cm^2

the perimeter $= 30.91 + 18.35 + 30.1 \simeq 79$ Cm

215 Graph the function f where $f(x) = \frac{1}{x} - 1$ 52
From the graph, find the domain and the range, then investigate its monotony and tell whether it is even, odd or otherwise.

Solution:



point of symmetry $(0, -1)$

Domain $= \mathbb{R} - \{0\}$

Range $= \mathbb{R} - \{-1\}$

the function is decreasing on each of $]-\infty, 0[$ and $]0, \infty[$

216 If $f(x) = 2^{x+1}$, find the solution set of:
(1) $f(x) = 32$ (2) $f(x-2) = \frac{1}{8}$

Solution:

(1) $2^{x+1} = 32 \Rightarrow 2^{x+1} = 2^5$

$\Rightarrow x + 1 = 5 \Rightarrow x = 4 \Rightarrow S.S = \{4\}$

(2) $2^{x-2+1} = \frac{1}{8}$

$\Rightarrow 2^{x-1} = 2^{-3}$

$\Rightarrow x - 1 = -3$

$\Rightarrow x = -2$

$S.S = \{-2\}$

217

Draw the graph of the function f:

$$f(x) = \begin{cases} x^2 & , x > 0 \\ -2x & , x < 0 \end{cases}$$

, then deduce its Range

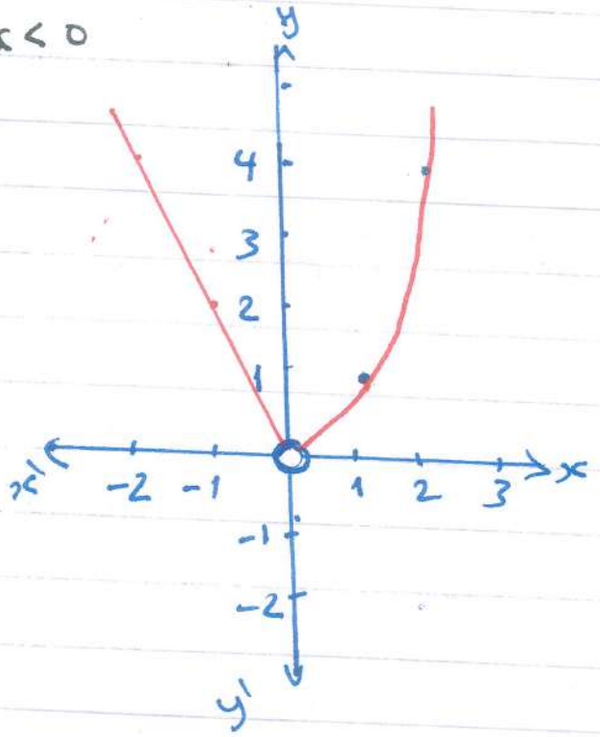
Solution

$f_1(x) = x^2$

x	0	1	2
$f(x)$	0	1	4

$-2x$

x	0	-1	-2
$f(x)$	0	2	4



Range $=]0, \infty[$

218 If $f_1 : \mathbb{R} \to \mathbb{R} : f_1(x) = 3x - 1$,

$f_2 : [-2, 3] \to \mathbb{R} : f_2(x) = 3 - 2x$,

graph $(f_1 + f_2)$, then deduce

① domain ② the monotonicity.

Solution:

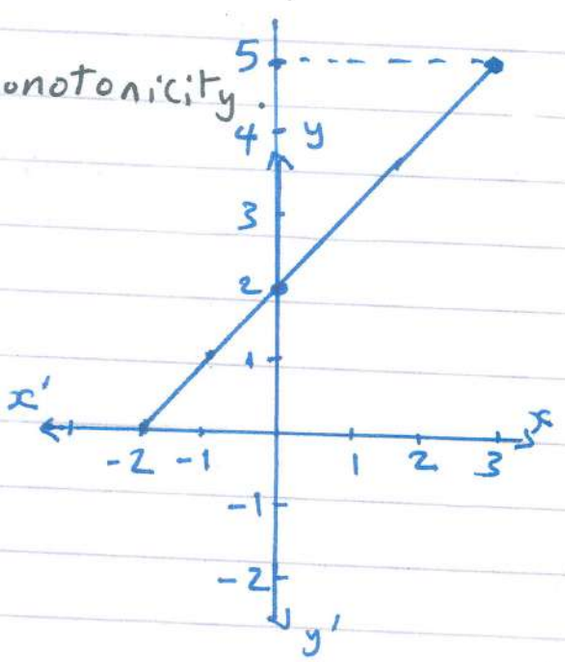
$(f_1 + f_2)(x) = x + 2$

Domain $= D_1 \cap D_2$

$= [-2, 3]$

the function is increasing on

$]-2, 3[$



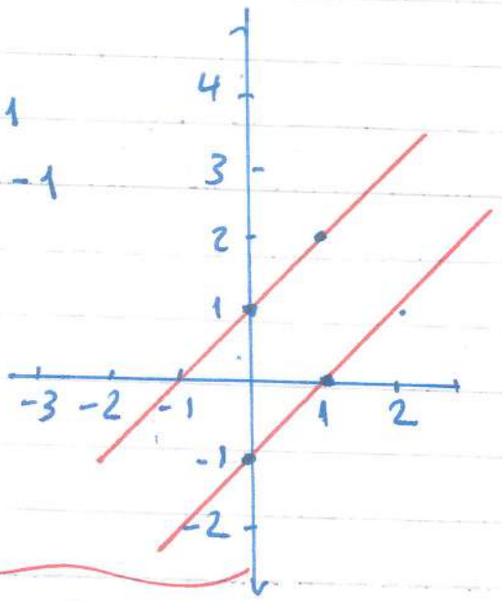
219 Find the inverse function of the function f:
$f(x) = x + 1$, then graph $f(x)$, $f^{-1}(x)$

Solution:

$\because y = x + 1 \quad \therefore x = y + 1$

$\Rightarrow y = x - 1$

$\therefore f^{-1}(x) = x - 1$



220 Solve in $\mathbb{R}$ the following two functions:

① $\log_4 x = 1 - \log_4 (x-3)$ ② $|x+2| = |x-3|$

Solution

[1] $\log_4 x + \log_4 (x-3) = 1$

$\Rightarrow \log_4 x(x-3) = 1$

$\Rightarrow x(x-3) = 4 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow (x+1)(x-4) = 0$

$\Rightarrow x = -1$ refused or $x = 4$

$S.S = \{4\}$

[2] $|x+2| = \pm(x-3)$

$x + 2 = x - 3$ | $x + 2 = -x + 3$

$2 \neq -3$ | $2x = 1$

$x = \frac{1}{2}$

$S.S = \{\frac{1}{2}\}$

221 Find (a) $\lim_{x \to 2} \frac{x^5 - 32}{x^2 + 3x - 10}$ (b) $\lim_{x \to 0} \frac{\sin 2x + 5 \sin 3x}{x}$

Solution:

(a) $\lim_{x \to 2} \frac{x^5 - 2^5}{(x-2)(x+5)} = \lim_{x \to 2} \frac{x^5 - 2^5}{x^1 - 2^1} \cdot \lim_{x \to 2} \frac{1}{x+5}$

$= \frac{5}{1}(2)^{5-1} \times \frac{1}{2+5} = \frac{80}{7}$

(b) $\lim_{x \to 0} \left(\frac{\sin 2x}{x} + \frac{5 \sin 3x}{x} \right) = 2 + 5 \times 3 = 17$

---

222 Solve the acute-angled triangle ABC in which $a = 21$ cm., $b = 25$ cm. and the length of the diameter of the circumcircle of the triangle ABC equals 28 cm.

Solution:

$a = 21$, $b = 25$ cm , $r = \frac{28}{2} = 14$ cm

from sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$

$\Rightarrow \frac{21}{\sin A} = \frac{25}{\sin B} = \frac{28}{1}$

$\Rightarrow \sin A = \frac{21}{28} \Rightarrow m(\angle A) = 48^\circ\ 35'\ 25.36''$

and $\sin B = \frac{25}{28} \Rightarrow m(\angle B) = 63^\circ\ 14'\ 4.20''$

$\Rightarrow m(\angle C) = 180 - (48^\circ\ 35'\ 25.36'' + 63^\circ\ 14'\ 4.20'')$

$= 68^\circ\ 10'\ 30.44''$

$$\Rightarrow \frac{C}{\sin 68^\circ 10' 30.44''} = 28 \text{ cm}$$

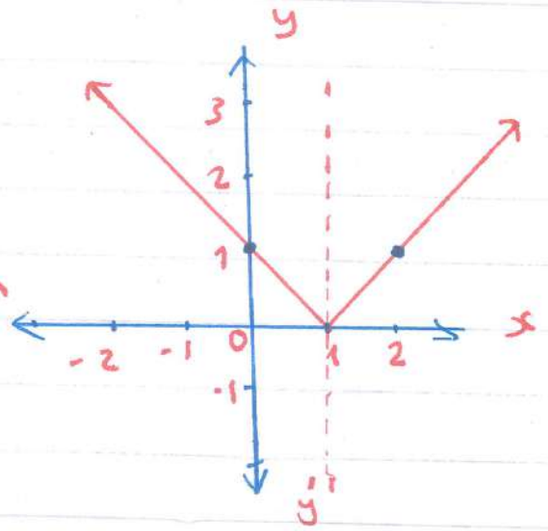
$$\Rightarrow C \simeq 26 \text{ cm}$$

223 From the opposite graph, find:

(1) $\lim_{x \to 1} f(x)$

(2) $\lim_{x \to 2} f(x)$

(3) $f(1)$



Solution: $f(x) = \begin{cases} x-1, & x \geq 1 \\ -x+1, & x < 1 \end{cases}$

(1) $\lim_{x \to 1} f(x)$

$\lim_{x \to 1^-} -x+1 = 0$, $\lim_{x \to 1^+} x-1 = 0$

$\Rightarrow \lim_{x \to 1} f(x) = 0$

(2) $\lim_{x \to 2} f(x) = \lim_{x \to 2} (x-1) = 2-1 = 1$

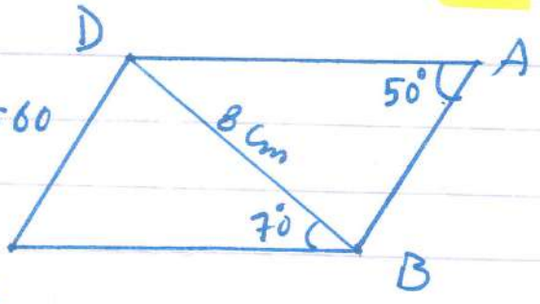
(3) $f(1) = 0$

224 ABCD is a parallelogram in which $m(\angle A) = 50^\circ$, $m(\angle DBC) = 70^\circ$, $BD = 8$ cm. Find the perimeter of the parallelogram.

57

$m(\angle C) = m(\angle A) = 50^\circ$

, $m(\angle CDB) = 180 - (50 + 70) = 60$



In triangle DBC

$$\frac{8}{\sin 50} = \frac{BC}{\sin 60} = \frac{DC}{\sin 70}$$

$$\Rightarrow BC = \frac{8 \sin 60}{\sin 50} \simeq 9 \text{ cm}$$

, $DC = \frac{8 \sin 70^\circ}{\sin 50^\circ} \simeq 9.8$ cm

, then the perimeter of the parallelogram =

$(DC + BC) \times 2 = (9 + 9.8) \times 2 \simeq 37.6$ cm

225 ABC is a triangle in which $a = 5$ cm, $b = 7$ cm, $m(\angle A) = 40^\circ$

Find: $m(\angle B)$.

Solution:

$a = 5$ cm, $b = 7$ cm , $m(\angle A) = 40$

From the Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{5}{\sin 40} = \frac{7}{\sin B}$$

$$\sin B = \frac{7 \sin 40^\circ}{5} \simeq 0.8999$$

$$\Rightarrow m(\angle B) = 64^\circ \ 8' \ 42.99''$$

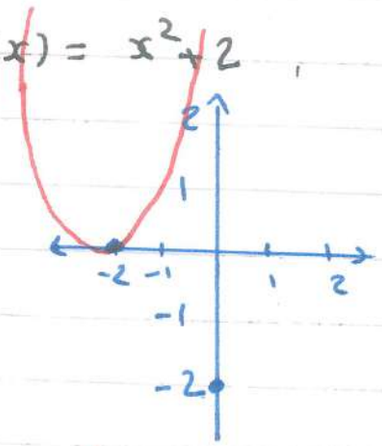
226 Use the curve of the function $f: f(x) = x^2$ to represent each of:

① $f_1(x) = f(x+2)$ ② $f_2(x) = x^2 + 2$

Solution:

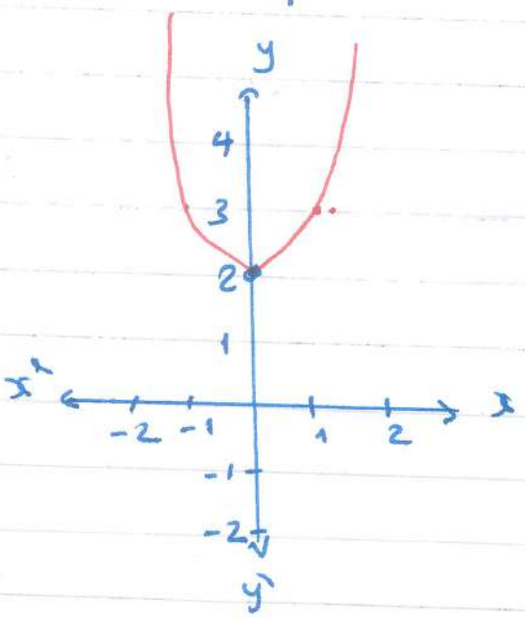
$f_1(x) = (x+2)^2$

the point of symmetry is $(-2, 0)$



$f_2(x) = x^2 + 2$

the point of symmetry is $(0, 2)$



227 Find in $\mathbb{R}$ the solution set of the inequality:

$|3x - 2| \geq 7$

Solution:

$x \geq \frac{2}{3}$

$3x - 2 \geq 7$

$3x \geq 9 \quad \div 3$

$x \geq 3$

$x < \frac{2}{3}$

$3x - 2 \leq -7$

$3x \leq -5 \quad \div 3$

$x \leq -\frac{5}{3}$

$S.S = \mathbb{R} -]-\frac{5}{3}, 3[$

228 Find the value of a which makes the function f continuous at $x = 2$

where $f(x) = \begin{cases} x^2 - 1 , & x \geq 2 \\ x - 2a , & x < 2 \end{cases}$

Solution:

Since f is continuous at $x = 2$

$\Rightarrow \quad f(2^-) = f(2^+)$

$(2)^2 - 1 = 2 - 2a$

$\Rightarrow \quad 3 = 2 - 2a \Rightarrow 2a = -1 \Rightarrow \boxed{a = -\frac{1}{2}}$

229 Discuss the existence of $\lim_{x \to 0} f(x)$ where

$$f(x) = \begin{cases} \dfrac{\tan 2x}{\sin x} , & x > 0 \\ \dfrac{5x + 6}{x + 3} , & x < 0 \end{cases}$$

$$f(0^-) = \lim_{x \to 0^-} \frac{5x + 6}{x + 3} = \frac{6}{3} = 2 \quad \ldots (1)$$

$$f(0^+) = \lim_{x \to 0^+} \frac{\frac{\tan 2x}{x}}{\frac{\sin x}{x}} = \frac{2}{1} = 2 \quad \ldots (2)$$

from 1, 2 $\quad \lim_{x \to 0} f(x) = 2$

230 Find in $\mathbb{R}$ the solution set of the equation:

$$x^{4/3} - 10\,x^{2/3} + 9 = 0$$

Solution:

$$(x^{2/3} - 1)(x^{2/3} - 9) = 0$$

$x^{2/3} = 1$ | $x^{2/3} = 9$

$x = 1^{3/2} = 1$ | $x = 9^{3/2}$

$x = \pm 27$

$S.S = \{1, 27, -27\}$

231 If: $f(x) = a^x$, prove that:

$$\frac{1}{f(x)+1} + \frac{1}{f(-x)+1}$$ has a constant value whatever the value of x

Solution:

$$\frac{1}{f(x)+1} + \frac{1}{f(-x)+1} = \frac{1}{a^x+1} + \frac{1}{a^{-x}+1}$$

$$= \frac{a^{-x}+1+a^x+1}{(a^x+1)(a^{-x}+1)} = \frac{a^x + a^{-x} + 2}{1 + a^x + a^{-x} + 1}$$

$$= \frac{a^x + a^{-x} + 2}{a^x + a^{-x} + 2} = 1 = \text{Constant}$$

232

Find: $\lim_{x \to 1} \frac{4 - \sqrt{x+15}}{1 - x^2}$

Solution:

$$L = \lim_{x \to 1} \frac{4 - \sqrt{x+15}}{1 - x^2} = \lim_{x \to 1} \frac{4 - \sqrt{x+15}}{1 - x^2} \times \frac{4 + \sqrt{x+15}}{4 + \sqrt{x+15}}$$

$$= \lim_{x \to 1} \frac{16 - (x+15)}{(1 - x^2)(4 + \sqrt{x+15})}$$

$$= \lim_{x \to 1} \frac{(1 - x)}{(1 - x)(1 + x)(4 + \sqrt{x+15})}$$

$$\Rightarrow \lim_{x \to 1} \frac{1}{(1+x)(4+\sqrt{x+15})} = \frac{1}{(2)(8)} = \frac{1}{16}$$

233 If the function f where

$$f(x) = \begin{cases} \frac{x^2 + 2x - 3}{x+3} & , x \neq -3 \\ x + a & , x = -3 \end{cases}$$

is continuous at $x = -3$, find the value of a

Solution:

$$\lim_{x \to -3} \frac{x^2 + 2x - 3}{x+3} = \lim_{x \to -3} \frac{(x-1)(x+3)}{x+3} = \lim_{x \to -3} (x - 1) = -4$$

Since the function is continuous $\therefore f(-3) = \lim_{x \to -3} f(x)$

$\Rightarrow -3 + a = -4 \Rightarrow \boxed{a = -1}$

234 put in the simplest form:

$$\log_b a^2 \times \log_c b^3 \times \log_a c$$

Solution:

the expression $= \frac{\log a^2}{\log b} \times \frac{\log b^3}{\log c} \times \frac{\log c}{\log a}$

$$= \frac{2\log a}{\log b} \times \frac{3\log b}{\log c} \times \frac{\log c}{\log a}$$

$$= 2 \times 3 = 6$$

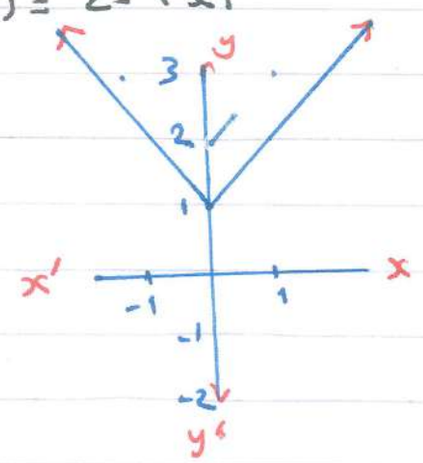
235 Use the curve of the function $f(x) = |x|$ to represent each of the following

① $f_1(x) = |x| + 1$ ② $f_2(x) = 2 - |x|$

Solution:

$f_1(x) = |x| + 1$

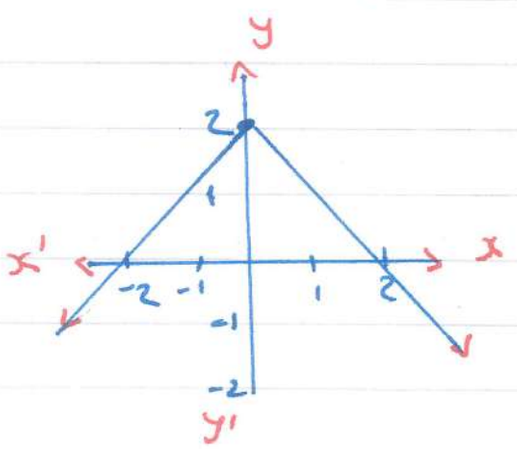
point of symmetry $(0, 1)$



$f_2(x) = 2 - |x|$

point of symmetry

$(0, 2)$



236 ABC is a triangle in which:

$\frac{1}{3}\sin A = \frac{1}{4}\sin B = \frac{1}{5}\sin C$, find : $m(\angle C)$, and if the perimeter of the triangle = 24 cm. find its surface area.

Solution:

$$\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{5}$$

$\Rightarrow a = 3k, \quad b = 4k, \quad c = 5k$

from the cosine rule:

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{9k^2+16k^2-25k^2}{2\times 3k\times 4k}$$

$$= \frac{0}{24k^2} = 0$$

$\Rightarrow m(\angle C) = 90^\circ$

$\because$ the perimeter $= 24$

$\Rightarrow 3k+4k+5k = 24$

$\Rightarrow 12k = 24 \Rightarrow k = 2$

$\Rightarrow a = 6$ cm, $b = 8$ cm, $c = 10$ cm

$\therefore$ the area $= \frac{1}{2}ab\sin C = \frac{1}{2}\times 6\times 8\times \sin 90$

$= 24 \text{ cm}^2$

237 If $\lim_{x \to 2} \frac{x^2 - 4a}{x-2}$ exists, then $a = \ldots$

(a) -1 (b) 1 (c) 2 (d) 4

238 $\lim_{x \to \infty} (4 + 3x - x^3) = \ldots$

(a) 4 (b) 2 (c) ∞ (d) $-\infty$

239 If $a < b <$ zero, then $\lim_{x \to \infty} \frac{x^a}{x^b} = \ldots$

(a) ∞ (b) $-\infty$ (c) zero (d) $a-b$

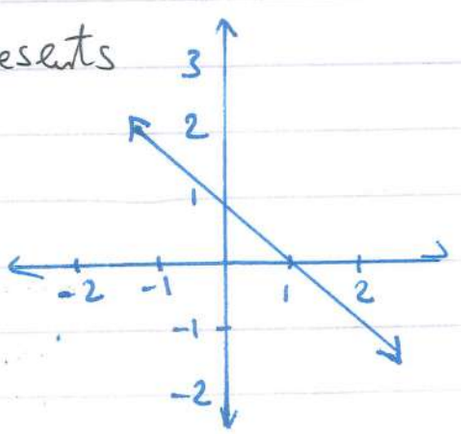
240 Discuss the continuity of the function f,

where $f(x) = \begin{cases} x^2 + 3 & , x \geq 1 \\ \frac{x^2 + 2x - 3}{x-1} & , x < 1 \end{cases}$

241 Investigate the existence of $\lim_{x \to 3} f(x)$

given that $f(x) = \begin{cases} \frac{x^2 - 7x + 12}{x-3} & , x > 3 \\ 2x - 7 & , x < 3 \end{cases}$

242 The opposite figure represents the curve of the function f, then find $\lim_{x \to 2} |f(x)|$



243 If $f(x) = 3x+1$, $g(x) = x^2 - 5$, then ...
$(g \circ f)(-3) = \cdots$
(a) -5 (b) 5 (c) 59 (d) -95

244 If f is an odd function on $[-s, s]$, then $f(-x) + f(x) = \cdots$
(a) $2x$ (b) undefined (c) $-2x$ (d) zero

245 The range of the function $f: f(x) = \frac{x-2}{2-x}$ equals ...
(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\{-1\}$

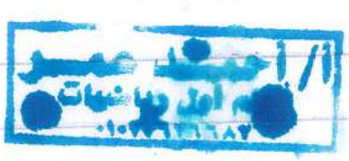
246 Graph the function $f: f(x) = \begin{cases} |x|, & x \leq 0 \\ x^2, & x > 0 \end{cases}$

from the graph state the range of the function and discuss its monotony, and its type whether it is odd, even or otherwise

247 If $f_1(x) = x^5$, $f_2(x) = \sin x$, find $(f_1 + f_2)$ hence find the type of $(f_1 + f_2)$ whether it is even, odd or otherwise

248 find the domain of the function

$f: f(x) = \frac{2x+1}{x-2}$ and prove that f is one-to-one.



249 Find

(1) $\lim_{x \to 0} \frac{x^2 + \sin 3x^2}{x \tan 2x}$

(2) $\lim_{x \to} \frac{\sqrt{x+4} - 2}{x^2 + x}$

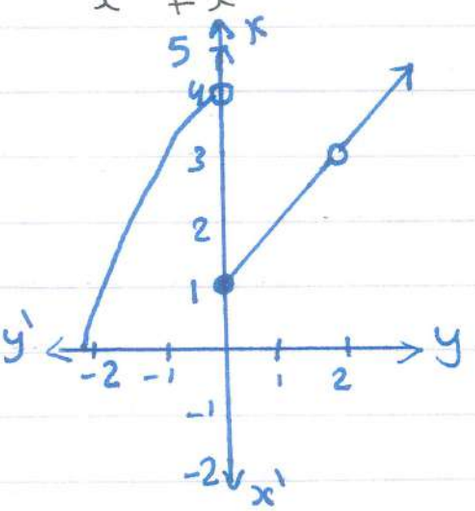
250 In the opposite figure, find:

(1) $f(\text{zero}^+)$

(2) $f(\text{zero}^-)$

(3) $f(2)$

(4) $\lim_{x \to 2} f(x)$



251 Find:

(1) $\lim_{x \to \infty} \frac{1}{x} \sqrt{3 + 4x^2}$

(2) $\lim_{x \to \infty} x \left(\sqrt{4x^2 + 1} - 2x \right)$

(3) $\lim_{x \to \infty} \frac{x^7 + 5}{3x^4 - 8}$

(4) $\lim_{x \to 4} \frac{(x-3)^7 - 1}{x - 4}$

(5) $\lim_{x \to 0} \frac{x^2 + \sin 3x^2}{x \tan 2x}$

(6) $\lim_{x \to 0} \frac{5 - 5\cos x}{x}$

252 If $f(x)=\frac{1}{x}$, $g(x)=x+3$, find:

(1) $(f\circ g)(x)$ (2) $(g\circ f)(x)$
and state the domain in each case.

253 Find in $\mathbb{R}$ the solution set of each of the following:

(1) $\sqrt{x^2-6x+9}+2x=9$

(2) $\frac{1}{|2x-3|}>2$

254 Graph the function $f: f(x)=\sqrt{x^2-4x+4}$ and determine its range and discuss its monotony.

255 Find algebraically the solution set of the equation

$$|x-3|=|9-2x|$$

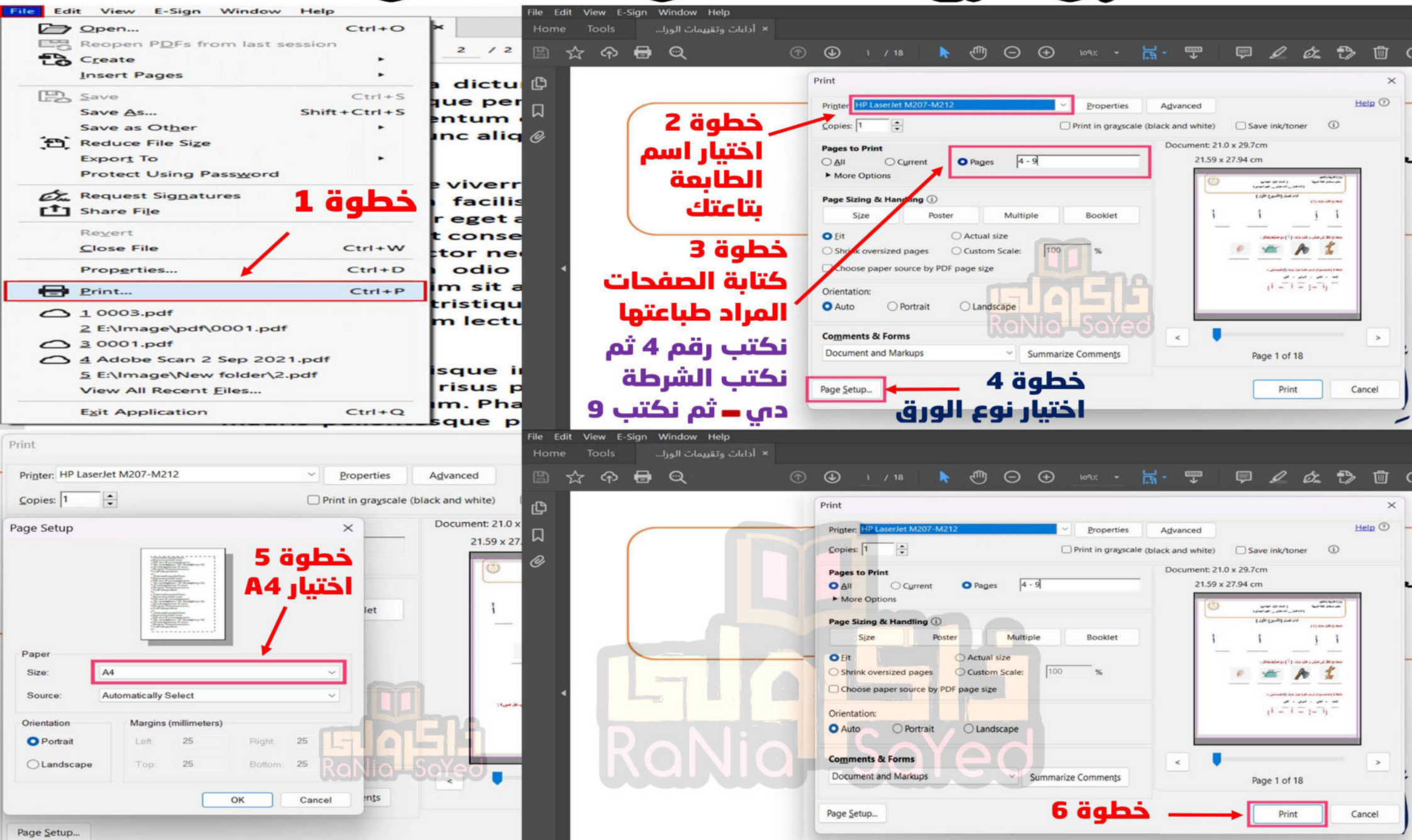
256 find the solution set of the inequality: $\sqrt{4x^2-12x+9}\leq 9$ in $\mathbb{R}$

with my best wishes

MR/ Ahmed omar

كيفية طباعة صفحات معينة من ملف معين

مثلا ازاى نطبع الصفحات من صفحة 4 الى صفحة 9



امتحانات رقم (1)

الترم الاول

عاشور لغة - لضاد

Model Exam of Second year secondary First Term 2023- 2024
General Mathematics **Time: 3 hours**

نموذج استرشادى رياضيات العامة للصف الثانى الثانوى أدبي للعام الدراسى ٢٠٢٣ / ٢٠٢٤م

First: Choose the correct answer

1) The domain of the function f : f $(x) = \frac{2x}{1-x^2}$ is .....

A $\{1, -1\}$ B $R - \{1\}$ C $R - \{1, -1\}$ D $\{2, 0, -1, 1\}$

2) $\lim\limits_{x \to 3} \left(\frac{x^2-3}{x-1}\right)$ = ...

A 3 B 6 C 0 D $\frac{1}{3}$

3) ABC is a triangle in which m $(\angle A) = 30^o$, m $(\angle C) = 60^o$, if c = $15\sqrt{3}$ cm : then a = ... cm

A 60 B 45 C 30 D 15

4) The curve of the function f : f $(x) = 2^{x+1}$ intersects Y-axis at the point .....

A (1 , 4) B (0 , 2) C (0 , 4) D (1 , 0)

5) Which of the following functions represents an even function ?

A f (x) = 2 x + 5 B g (x) = x sin x C h (x) = $2x^2 - x$ D n (x) = x cos x

6) 1) The measure of the greatest angle in the triangle whose sides length are: 3 cm , 5 cm and 7 cm is ..... ^{o}

A 150 B 110 C 120 D 100

7) $\lim\limits_{x \to 4} \left(\frac{4x-16}{x^2-16}\right)$ = ...

A $\frac{1}{4}$ B $\frac{1}{2}$ C 2 D 4

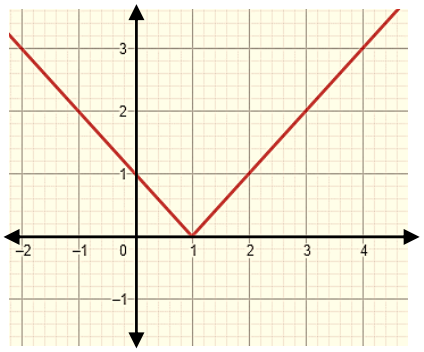
8) $\lim\limits_{x \to -1} \left(\frac{4x+4}{x+1}\right)$ = ...

A – 1 B 1 C 2 D 4

9) *In the opposite figure* :

The range of the function f :

f (x) = | $x - 1$ | is .....



A] – ∞ , 1] B [1 , ∞ [C [0 , ∞ [D [0 , ∞]

10) The solution set of the equation : $\log_2 x = 4$ in R is .....

A { 8 } B { 2 } C { 16 } D { 4 }

11) The solution set of the inequality | $x - 2$ | < 6 in R is .....

A]– 4 , 8[B [– 4 , 6[C [2 , – 1[D [– 4 , 8]

12) In Δ ABC , if m (∠ C) = 60 °, we get $a^2 + b^2 - c^2 = k\,a\,b$, then k = ...

A $\frac{1}{2}$ B 2 C 1 D – 1

13) $\lim\limits_{x \to 2} \left(\frac{x^5 - 32}{x^3 - 8} \right)$ = ...

A $\frac{20}{3}$ B $\frac{5}{3}$ C 4 D 2

14) If $\lim\limits_{x \to 2} \left(\frac{3x - a}{x + 1} \right) = 1$, then a = ...

A 0 B 3 C 6 D 9

15) The solution set for the equation: | $3 - x$ | – 5 = 3 in R is .....

A {5 , – 11} B {– 5 , 11} C {8 , 5} D {11 , 8}

16) ABC is a triangle : if a = 7 cm , b = 9 cm , m (∠ C) = 30 °,
then its area = ... cm^2

A $\frac{63}{2}$ B $\frac{63}{4}$ C 63 D $\frac{63}{6}$

17) The axis of symmtery of the curve of the function f : f $(x) = (x - 1)^2 + 3$ is .....

A $x = 1$ B $x = -1$ C $x = 3$ D $x = -3$

18) In Δ ABC if a : sin A = 14 : 1 , then the circumference of the circumcircle of Δ ABC = ... unit length

A 14π B 7π C 28π D 49π

19) $\lim_{x \to \infty} \left(\frac{1 - 7x + 2x^2}{3x^2 + 1} \right)$ = ...

A $\frac{4}{3}$ B $-\frac{7}{3}$ C $\frac{2}{3}$ D $\frac{7}{3}$

20) If f : f $(x) = 3^x$, then the value of x which satisfies the equation f $(x - 1) = 81$ is .....

A 4 B 5 C 6 D 9

21) $\lim_{x \to 0} \left(\frac{(2 - 3x)^7 - 128}{16x} \right)$ = ...

A 16 B – 32 C – 41 D – 84

22) Domain of the function f : f $(x) = \log_3 (x - 1)$ is .....

A $]-\infty , 1[$ B $]0 , 1[$ C $]1 , \infty[$ D $[0 , \infty[$

23) $\lim_{x \to \infty} (7)^{\frac{1}{x}}$ = ...

A 7 B 1 C $\frac{1}{7}$ D 0

24) The point of symmetry of the curve of the function f : f $(x) = \frac{1}{x - 1} + 2$ is .....

A (1 , 2) B (2 , 1) C (– 1 , 2) D (1 , – 2)

25) $\frac{\log(3)^x}{\log(9)^x}$ = ...

A $\frac{1}{3}$ B $\frac{x}{3}$ C 2 D $\frac{1}{2}$

26) ABC is a triangle: if b = 4 cm , c = 5 cm , cos A = $\frac{2}{5}$, then a = ... cm

A 5 B 6 C 4 D 8

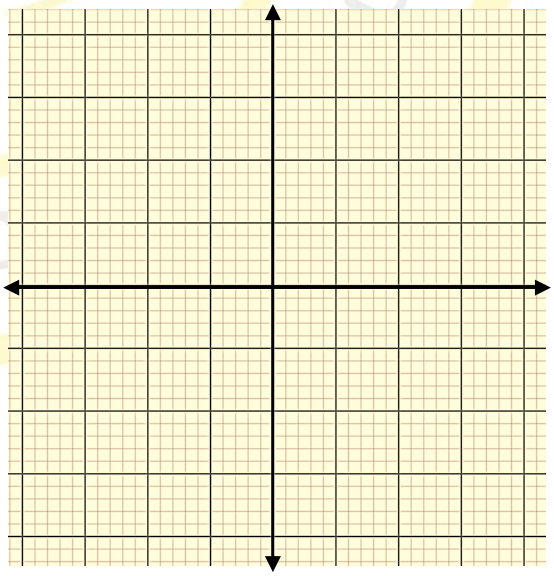
27) If $3^{x+1} = 17$, then x = ... (to the nearest one decimal number)

A 2.6 B 3.6 C 1.8 D 1.6

Second: Answer the following qestions:

1) Find : $\lim_{x \to 3} \left(\frac{2x^2 - 5x - 3}{x^2 - 9} \right)$

2) Draw the curve of the function f : f (x) = 2 – (x + 1)^2 , and from the graph find its range and discuss its monotony.



Model Answers of Second year secondary First Term 2023- 2024
General Mathematics (Arts section)

نموذج إجابة اختبار استرشادي نهاية الفصل الدراسي الأول الصف الثاني الثانوي (ادبى)
المادة: رياضيات عامة **2023 / 2024م**

First: Choose the correct answer

27 × 1 = 27 Marks

Question number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Anwser	C	A	D	B	B	C	B	D	C	C	A	C	A	B
Question number	15	16	17	18	19	20	21	22	23	24	25	26	27	
Anwser	B	B	A	A	C	B	D	C	B	A	D	A	D	

Second: Answer the following qestions:

2 Marks

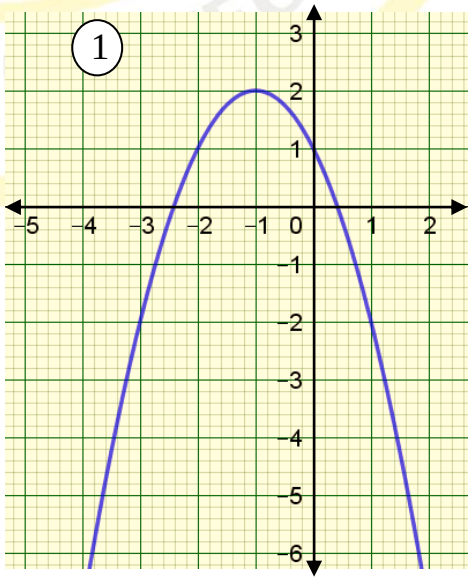
1) $\lim_{x\to 3}\left(\frac{2x^2-5x-3}{x^2-9}\right)=\lim_{x\to 3}\left(\frac{(2x+1)(x-3)}{(x+3)(x-3)}\right)$ (1)

$=\lim_{x\to 3}\left(\frac{(2x+1)}{(x+3)}\right)=\frac{7}{6}$ ($\frac{1}{2}$) ($\frac{1}{2}$)

3 Marks

2)

- The range = $]-\infty, 2]$ (1)
- Increasing when $x \in]-\infty, -1[$ ($\frac{1}{2}$)
- Decreasing when $x \in]-1, -\infty[$ ($\frac{1}{2}$)

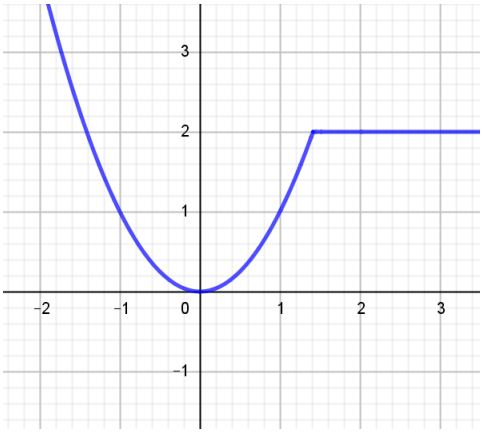


أسئلة استرشادية للصف الثانى الثانوى

رياضيات(١) للقسم الأدبى باللغة الإنجليزية

1-The opposite figure represents the graph of a function

The range of the function is …….



a) [0 , ∞ [

b) [0 , 2 [

c)] - ∞ , ∞ [

d)] - ∞ , 2 [

2- Which of the following relations represents a function?

a) $x + y^2 = 3$

b) $x^2 + y = 8$

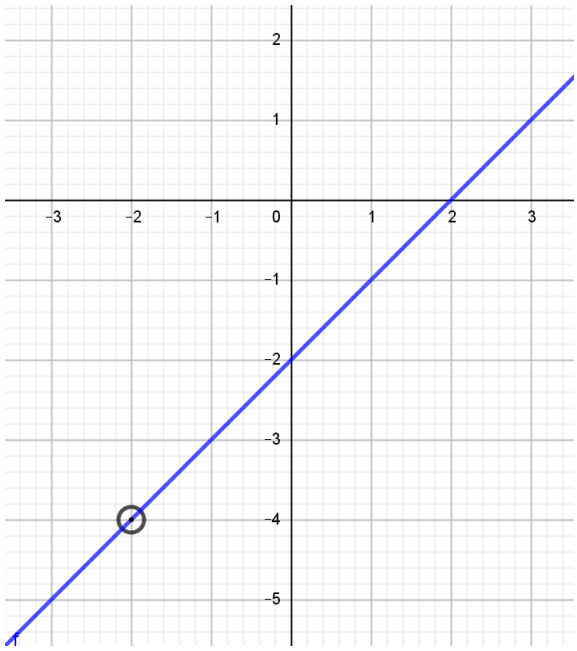
c) $x^2 + y^2 = 25$

d) $x = 5$

3- The opposite graph represents the function

$f(x) = \frac{x^2 - 4}{x+2}$

$\lim_{x \to -2} f(x)$ ....



a) Undefined

b) = 4

c) = - 4

d) = 2

4- In the triangle ABC the expression $\frac{b^2 + c^2 - a^2}{bc}$ = ...

a) cos a

b) 2cos a

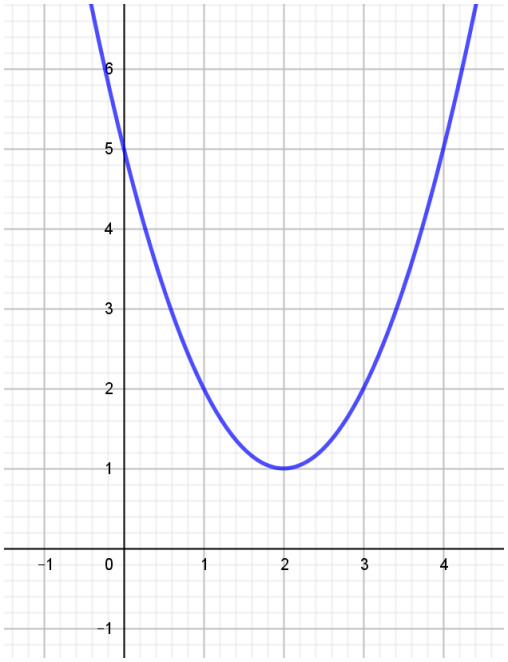
c) sin a

d) 2sin a

5- Find the solution set of $|x - 5| + 5 = x$.

6- In the opposite figure

$\lim_{x \to 2} f(x)$ ....



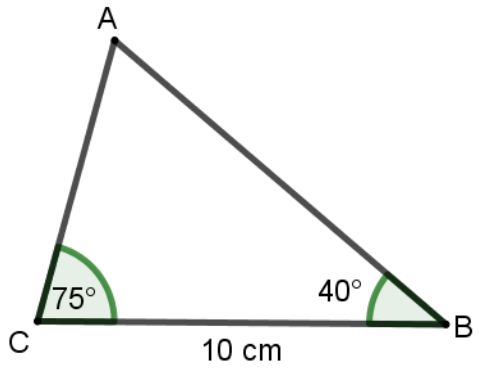
a) = 2
b) =5
c) =1
d) Not exist

7- If F: $R^+ \rightarrow R$, f(x) = x – 5 and n : [-1 , 5] → R , n(x) = x - 2,

Then find the domain of the function (f + n)(x).

8- In the opposite figure:

c = …… cm



a) 7

b) 10

c) 11

d) 8

9- Find $\lim_{x\to\infty} \frac{\sqrt{x^3+5x+7}}{x^2+4}$

10-In the triangle ABC,

If a = 7cm , m($\hat{B}$) =30° , m($\hat{C}$) =105°

Then b = …….. cm

a) $\frac{7}{2}$

b) $\frac{7\sqrt{2}}{2}$

c) 7

d) $7\sqrt{2}$

11- The solution set of the inequality:

$|x| + 2 <$ zero in R is…….

a) {-2}

b) {2}

c) ϕ

d)] -2 , 2 [

12- $\lim_{x \to 3} \frac{3x^4 - 243}{x - 3} = \cdots$

a) 81

b) 324

c) 4

d) 576

كيفية طباعة صفحات معينة من ملف معين

مثلا ازاى نطبع الصفحات من صفحة 4 الى صفحة 9